ON SELECTION OF PROBABILITY DISTRIBUTIONS FOR ANNUAL EXTREME RAINFALL SERIES IN EGYPT

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ABSTRACT

This paper presents an assessment procedure for evaluating the performance of various probability distributions in order to identify the most suitable model that could provide accurate extreme rainfall estimates in Egypt. More specifically, six popular probability distributions, Normal (N), Log-Normal (LN), Gumbel (EV1), Pearson Type III (P3), Log-Pearson Type III (LPIII), and Generalized extreme value (GEV), were examined and compared for their abilities in the estimation of annual maximum rainfalls. The suggested procedure was applied to annual maximum daily rainfall data from a network of 30 stations located in Egypt. To estimate the parameters of these distributions, two methods were used: the method of moment and L-moments. Based on numerical and graphical goodness-of-fit criteria, results indicated that the Log-Normal and Log-Pearson Type III distributions along with the L-moments are the best models for describing the distribution of daily annual maximum rainfalls in most stations in Egypt.

Keywords: Extreme Rainfall,Rainfall Frequency analysis;Probability distributions; L-moments;Egypt.

1 INTRODUCTION

The design and construction of certain projects, such as dams and urban drainage systems, the management of water resources, and the prevention of flood damage require an accurate estimation of extreme rainfall events. Rainfall Frequency Analysis (RFA) would enable us to determine the expected extreme rainfall for a given return period (Vivekanandan, 2014). Generally, extreme rainfall estimation models are based on annual maximum series (AMS) due to its simple structure. The most important step in RFA is to select a suitable probability distribution that could describe well the distribution of the annual maximum rainfall data. Yet, this step remains one of the major challenges in hydrologic practice due to spatial and temporal variability of rainfall data (Nguyen et al., 2017). The selection of an appropriate model depends mainly on the charactristics of available rainfall data at the particular site (Tao et al., 2002).

In general, several probability models have been used for describing the distribution of extreme rainfall data at a gauged site (e.g., Papalexiou and Koutsoyiannis, 2013; Gado, 2017; Nguyen et al., 2017). Nguyen and Mayabi (1991) studied the maximum daily rainfall in nine stations in the Montreal region (Canada) by using four probability distributions. The maximum likelihood(ML) method was used to estimate the parameters of the distributions. Based on the Chi-square and Akaike information criteria (AIC), the mixed exponential was found to be the best model.Tao et al. (2002)examined and compared nine popular probability distributions for their descriptive and predictive abilities in the estimation of annual maximum precipitations in southern Quebec, Canada. They used the methods of maximum likelihood and L-moments to estimate the parameters of these distributions. The Wakeby, GEV, and GNO models were proved to be the best models in their case study. However, they recommended the GEV distribution because it requires a simpler parameter estimation method and it isbased on a more solid theoretical basis for representing the distribution of extreme random variables. In a recent study, Nguyen et al. (2017) proposed a general procedure for assessing the performance of ten commonly used probability distributions in rainfall frequency analyses based on their descriptive and predictive abilities. The procedure was applied on 21 stations located in Ontario, Canada. The results indicated that the GEV, GNO, and PE3 models were the best models for describing the distribution of daily and sub-daily annual maximum rainfalls in the case study.

For Malaysia,Zalina et al.(2002) indicated that the GEV distribution is the most appropriate distribution for describing the annual maximum rainfall series based on somegoodness-of-fit criteria. Topaloglu(2002) applied some popular probability distributions to series of annual instantaneous flood peaks and annual peak daily precipitation in the Seyhan basin, Turkey. The methods of moments (MOM) and probability weighted moments (PWM) were used to estimate the parameters of the distributions. According to the evaluations of chi-squared tests, the Gumbel (MOM) was determined to be the best modelfor both flow and precipitation stations. For the K-S test, the Log-Normal-3 (MOM) and the Log-Pearson-3 (MOM) were found to be the best models for flow and precipitation stations, respectively.Generalized Exponential Distribution (GED) was recommended as the best distribution to describe the maximum rainfall series in the upper Lusatian Neisse River basin, Poland(Wdowikowski et al., 2016).

In Egypt, very few studies have analyzed extreme rainfall events (e.g., Gado, 2017). As far as this, non of the previous publications have addressed the comparison of diverse probability distributions based on different parameter estimation methods. Thus, the aim of this study is to compare some popular distributions along with different parameter estimation methods in order to find the most suitable model for describing the annual maximum daily rainfall data for 30 stations located in different regions in Egypt.

2 METHODOLOGY

2.1 Data

A network of 30 stations located in Egypt were selected for this study. The database was taken from a previous study (Gado, 2017). Some of characteristics of the selected stations used in this study are presented in Table 1. The record lengths for these datasets vary from 15 years (Kharga) to 55 years (Alexanderia Intl).

Table 1. Summary of characteristics of the selected stations in this study (Modified after Gado, 2017)

Station ID	Station name	Latitude	Longitude	Elevation (m)	Period of record	Record length (years)
62337	Al-Arish Intl	31.07	33.84	36.9	1985-2017	33
62318	Alexanderia Intl	31.18	29.95	-1.8	1957-2017	55
62414	Aswan Intl	23.97	32.78	200	1962-2017	22
62393	Asyut	27.04	31.01	226	1960-2016	20
62420	Baharia	28.33	28.90	130	1957-2017	25
62325	Baltim	31.55	31.08	2	1994-2017	24
147728	Cairo Ezbekia	30.05	31.25	20	1909-1957	46
62366	Cairo Intl	30.12	31.41	75	1957-2017	53
62309	Dabaa	31.03	28.44	18	1963-2017	23
	El-Tahmed	29.30	34.30	625	1922-1955	31
62459	Eltor	28.21	33.65	35	1961-2017	24
62423	Farafra	27.05	27.98	92	1957-2016	17
62375	Giza Cairo	30.03	31.21	28	1924-1957	31
147730	Helwan Cairo	29.86	31.34	116	1904-1957	54
62463	Hurghada	27.18	33.80	14	1991-2017	18
62440	Ismailia	30.59	32.25	13	1987-2017	28

62435	Kharga	25.46	30.53	73	1961-2014	15
62465	Kosseir	26.14	34.26	11	1960-2014	26
62405	Luxor	25.67	32.71	99	1957-2016	33
62306	MarsaMatrooh	31.33	27.22	30	1957-2017	51
62387	Minya	28.08	30.73	37	1957-2016	32
62452	Nekhel	29.91	33.74	403	2001-2017	17
62333	Port Said	31.28	32.24	2	1957-2017	24
62332	Elgamil	31.28	32.24	6	1987-2014	28
62455	RasSedr	29.58	32.72	16	2000-2017	18
62300	Salloum	31.53	25.18	6	1957-1994	26
62305	SalloumPlataou	31.57	25.13	6	1996-2017	21
62417	Siwa	29.20	25.48	-12	1957-2017	37
623664	ST Catherin Intl	28.69	34.06	1331	1934-2006	31
62357	WadiElnatroon	30.40	30.36	1	1996-2017	19

2.2 Probability Distributions

Six probability models were investigated in this study, namely, Normal (N), Log-Normal (LN), Pearson Type III (PIII), Log-Pearson Type III (LPIII), Gumbel (EV1), and Generalized Extreme Value (GEV) distributions. All these models have been commonly used in practice for representing the distribution of rainfall extremes (e.g., Tao et al., 2002; Zalina *et al.*, 2002; Olofintoye et al., 2013; Vivekanandan, 2014; Nguyen et al., 2017). Table 2 presents the probability density function *f* (*x*) and the associated parameters for each of these models(Rao and Hamed, 2000).

Table 2. The probability density function *f* (*x*) and the associated parameters for each distribution

Distribution	**probability density function**	**Parameters**		
		Location (α)	**Scale (β)**	**Shape (*k*)**
N	$f(x) = \frac{1}{\beta\sqrt{2\Pi}} * e^{-\frac{1}{2\beta^2}(x-\alpha)^2}$ -∞ < x < ∞	√	√	
LN	$f(x) = \frac{1}{\beta\sqrt{2\Pi}} * e^{-\frac{1}{2\beta^2}(y-\alpha)^2}$ y = log x , x > 0	√	√	
PIII	$f(x) = \frac{1}{\alpha\Gamma(k)}(\frac{x-\beta}{\alpha})^{k-1} * e^{-(\frac{x-\beta}{\alpha})}$ β < x < ∞	√	√	√
LPIII	$f(x) = \frac{1}{\alpha\Gamma(k)}(\frac{x-\beta}{\alpha})^{k-1} * e^{-(\frac{y-\beta}{\alpha})}$ y = log x	√	√	√
EV1	$f(x) = \frac{1}{\beta} e^{\left[-\left(\frac{x-\alpha}{\beta}\right)-e^{-\left(\frac{x-\alpha}{\beta}\right)}\right]}$ -∞ < x < ∞	√	√	
GEV	$f(x) = \frac{1}{\beta}\left[1-k(\frac{x-\alpha}{\beta})\right]^{\frac{1}{k}-1} * e^{-\left[1-k\left(\frac{x-\alpha}{\beta}\right)\right]^{1/k}}$	√	√	√

2.3 Parameter Estimation Methods

A number of methods can be used for parameter estimation in application to hydrologic extremes, such as (Rao and Hamed, 2000): the method of moments (MOM), the maximum likelihood method (MLM), the probability weighted moments method (PWM), the least squares method (LS), maximum entropy (ENT), mixed moments (MIX), the generalized method of moments (GMM), and incompletemeans method (ICM). Two of the very commonly used methods are considered in the present study, namely, the method of moments (MOM) and L-moments method. The method of L-moments is considered one of the most popular parameter estimation methods because of its computational simplicity and good performance for small samples (Gado and Nguyen, 2016).

2.4 Goodness of Fit

Various graphical and numerical criteria could be employed to evaluate the performance of different distributions in describing the extreme rainfall data and in extrapolating quantiles that lie beyond the available record length in order to identify the best distribution(s) (Nguyen *et al.*, 2017). In the present study,the quantile-quantile (Q – Q) plots were used to compare the observed and the estimated values.In addition,seven numerical criteria were used for a more objective assessement of the different distributions.These criteria are: Root Mean Square Error (RMSE), Relative Root Mean Square Error (RRMSE), MaximumAbsolute Error (MAE), Correlation Coefficient (CC), BIAS, BIASr, and Akike Information Criteria (AIC). Formulas for the seven criteria are as given below:

$$\text{RMSE} =[\sum (x_i - y_i)^2/(n-m)]^{1/2} \quad (1)$$

$$\text{RRMSE} = [\sum ((x_i - y_i)/x_i)^2/(n-m)]^{1/2} \quad (2)$$

$$\text{MAE} = \max |x_i - y_i| \quad (3)$$

$$\text{CC} = \sum(x_i - \bar{x})(y_i - \bar{y})/[\sum(x_i - \bar{x})^2 \sum(y_i - \bar{y})^2]^{1/2} \quad (4)$$

$$\text{BIAS} = (1/n)\sum_{i=1}^{n}(x_i - y_i) \quad (5)$$

$$\text{BIASr} = (1/n)\sum_{i=1}^{n}((x_i - y_i)/x_i) \quad (6)$$

$$\text{AIC} = 2m + n\ln(\text{RSS}/n) \quad (7)$$

Where x is the observed value; y is the value computed from the assumed probability distribution; n is the total number of observations; m is the number of parameters estimated for the assumed distribution; and RSS is the residual sum of squares.

3 RESULTS

In this study, six probability distributions were considered (N, LN, PIII, LPIII, EV1, and GEV) along with two parameter estimation methods (MOM and L-moments).Furthermore, two types of goodness of fit were used: graphical and numerical criteria (RMSE, RRMSE, MAE, CC, BIAS, BIASr, and AIC). The Q-Q plots of all 30 AMS show that all distributions closely described the left-tail and central parts. The focus is on the right tail region of the distribution since this is the region of importance to engineering design and planning applications. The right-tail parts, however, are less well described and there are no obvious trends. These values can be well estimated, over-estimated, or under-estimated by any of the different models. In order to select the best model for every station, the different distributions were compared by using: the MOM, the L-moments method, and both methods.

In the case of MOM, the numerical criteria showed that the Log-Pearson Type III and the Log-Normal are the best distributions for representing the AMS of most stations in the case study. Table 3 presents values of the parameters of the best distribution for each station. For purposes of illustration, the fitting of the six distributions of the AMS by the MOM at Al-Arish station is presented in Fig. 1.The number of stations for each selected best distribution using the MOM is presented in Fig. 3-a.

In the case of L-moments, the log-Normal distribution is considered the best model at most stations in Egypt, according to the goodness-of-fit criteria. The values of the parameters of the best distribution for each station are shown in Table 4. Fig. 2 presents the fitting of the six distributions of the AMS by the L-moments method at Al-Arish station.Fig. 3-b shows the number of stations for each selected best distribution using the L-moments.

To select the optimum model for each station that contains the best distribution along with the best parameter estimation method, the results of both methods were compared. It can be concluded that the Log-Normal distribution with the L-moments is the optimum model for most stations in Egypt(Fig. 3-c). Then, the Log-Pearson Type III distribution with the L-moments is considered the second best model in Egypt. Fig. 3-d presents the number of stations for each selected best parameter estimation method. It can be concluded that the L-moments method is better than the MOM for estimating the parameters for extreme rainfall events in Egypt.

The purpose of selecting the optimum model for each station is to accurately predict extreme rainfall depths for different return periods.Therefore, the selected best models for each station were used to estimate the quantiles for return periods of 2, 5, 10, 25, 50, 100, 200 and 1000 years (Table 5).To get reliable predictions of quantile extreme rainfall, it is known to restrict extrapolation of such quantiles to return periods of up to twice as long as the record length (Natural Research Council of Canada, 1989). Therefore, for the selected rainfall records with a length ranging from 15 to 55 years, the extreme rainfall estimates for up to 100 year could be considered reasonably accurate, whereas the 200-year and 1000-year estimates would be less representative.

Fig. 4 shows the 10-year and 100-year daily extreme rainfall estimation for the selected stations. It can be shown that the 10-year daily extreme rainfall varies from 6 mm at Baharia to 83 mm at Hurghada(Fig. 4-a). For the return period of 100 year, the daily extreme rainfall estimation ranges from 17 mm at Baharia to 281 mm at Port Said Elgamil(Fig. 4-b).

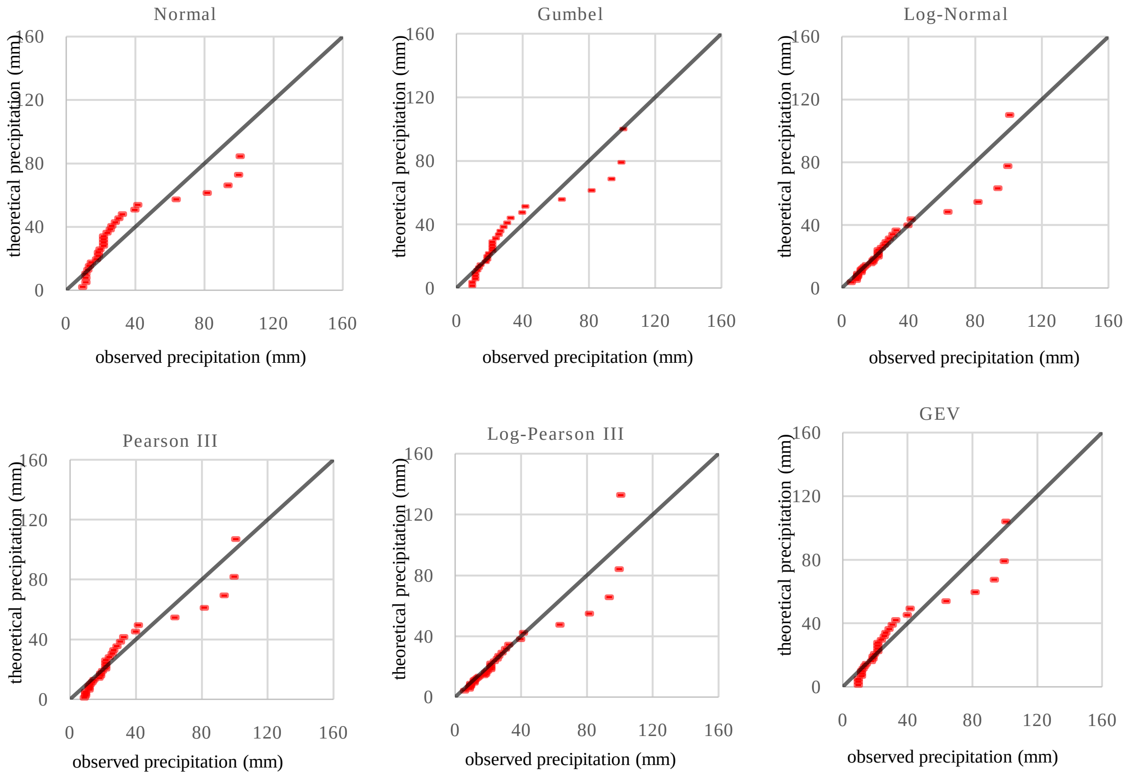


Figure 1. Q-Q plots for the six distributions fitted to daily AMS by using the MOM at Al-Arishstation

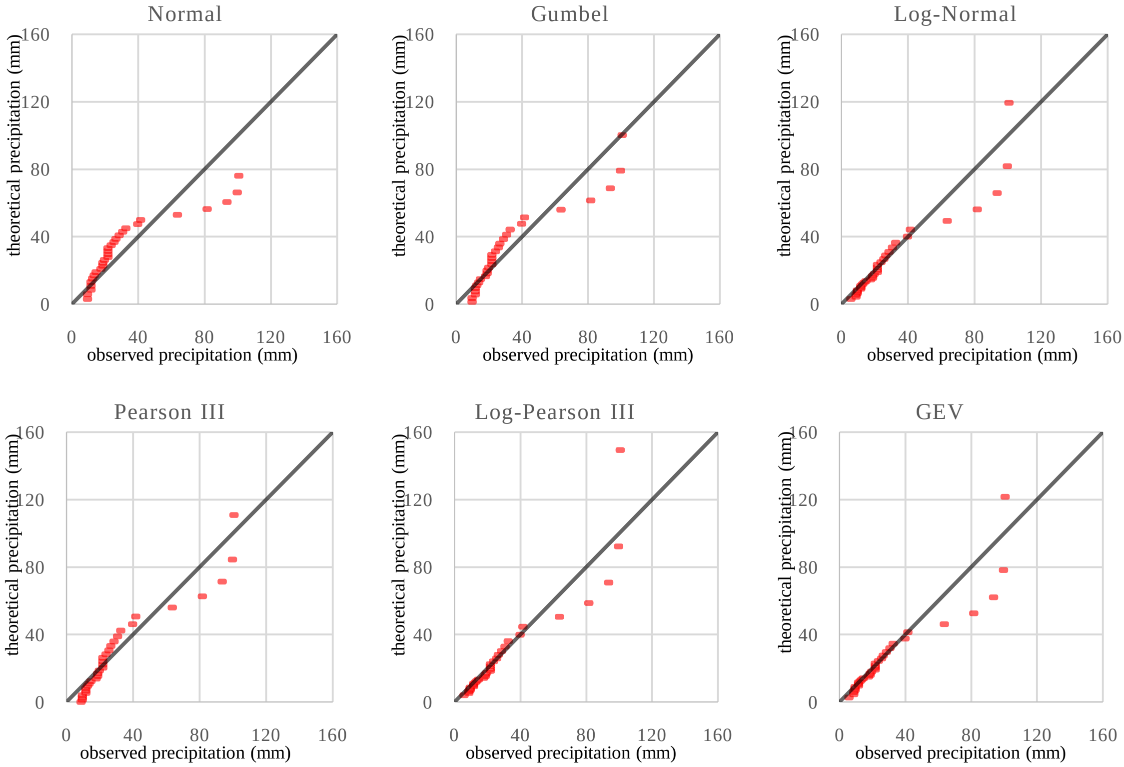


Figure 2. Q-Q plots for the six distributions fitted to daily AMS by using the L-moments at Al-Arishstation

Table 3. Values of the parameters of the best models for each station by using the MOM

Station	Distribution	Parameters		
		Location(α)	Scale(β)	Shape(k)
Al-Arish Intl	LOGN	3.001	0.812	-
Alexanderia Intl	LOGPIII	0.072	-2.052	73.857
Aswan Intl	LOGPIII	0.868	- 0.815	1.719
Asyut	LOGPIII	0.280	-5.765	24.721
Baharia	LOGPIII	0.068	-14.202	217.757
Baltim	LOGN	2.948	0.763	-
Cairo Ezbekia	LOGN	2.025	0.750	-
Cairo Intl	PIII	26.583	-7.755	1.130
Dabaa	LOGN	2.935	0.702	-
El-Tahmed	LOGPIII	0.345	- 0.584	8.466
Eltor	LOGPIII	0.277	- 4.331	22.336
Farafra	LOGPIII	0.205	-10.675	58.636
Giza Cairo	LOGPIII	0.132	- 3.827	43.608
Helwan Cairo	LOGPIII	0.175	-1.478	20.862
Hurghada	EV1	29.519	18.598	-
Ismailia	LOGN	2.398	1.079	-
Kharga	LOGN	1.500	1.115	-
Kosseir	LOGPIII	0.247	- 6.513	33.286
Luxor	LOGN	2.189	1.098	-
MarsaMatrooh	LOGPIII	0.259	- 0.227	12.290
Minya	LOGN	2.034	1.218	-
Nekhel	LOGPIII	0.462	- 0.465	5.598
Port Said	LOGPIII	-0.617	0.265	11.021
Port Said Elgamil	LOGN	2.816	1.177	-
RasSedr	LOGN	1.263	1.340	-
Salloum	LOGN	2.476	0.809	-
SalloumPlataou	LOGN	2.751	0.874	-
Siwa	LOGN	0.978	1.383	-
ST Catherin Intl	LOGPIII	-5.611	0.221	30.260
WadiElnatroon	LOGPIII	0.067	- 15.490	260.186
Al-Arish Intl	LOGN	2.852	0.951	-

Table 4. Values of the parameters of the best models for each station by using the L-moments

Station	**Distribution**	**Parameters**		
		Location(α)	**Scale(β)**	**Shape(*k*)**
Al-Arish Intl	LOGN	2.948	0.875	-
Alexanderia Intl	GEV	20.832	10.670	- 0.316
Aswan Intl	LOGPIII	1.015	- 0.592	1.250
Asyut	LOGN	1.132	1.368	-
Baharia	LOGN	0.636	0.940	-
Baltim	LOGN	2.922	0.796	-
Cairo Ezbekia	GEV	14.477	9.935	- 0.355
Cairo Intl	LOGN	1.992	0.794	-
Dabaa	PIII	46.917	2.949	0.412
El-Tahmed	LOGN	2.956	0.672	-
Eltor	LOGPIII	0.493	0.178	4.375
Farafra	LOGN	1.848	1.308	-
Giza Cairo	LOGN	1.427	1.414	-
Helwan Cairo	LOGPIII	0.187	- 2.323	22.687
Hurghada	PIII	8.009	1.535	1.264
Ismailia	GEV	7.308	5.410	- 0.189
Kharga	GEV	16.199	23.576	- 0.202
Kosseir	LOGN	2.067	1.351	-
Luxor	LOGN	1.169	1.380	-
MarsaMatrooh	LOGN	1.664	1.484	-
Minya	LOGN	1.783	1.420	-
Nekhel	PIII	28.106	5.349	0.781
Port Said	LOGN	1.514	1.589	-
Port Said Elgamil	LOGPIII	0.764	0.383	2.277
RasSedr	GEV	8.159	6.357	- 0.244
Salloum	LOGN	2.316	0.788	-
SalloumPlatao u	LOGN	2.725	1.252	-
Siwa	LOGN	1.309	1.305	-
ST Catherin Intl	LOGN	2.401	0.897	-
WadiElnatroon	LOGN	2.637	0.996	-
Al-Arish Intl	GEV	2.009	2.114	- 0.642
Al-Arish Intl	LOGN	2.029	0.991	-
Alexanderia Intl	PIII	46.780	- 0.432	0.591

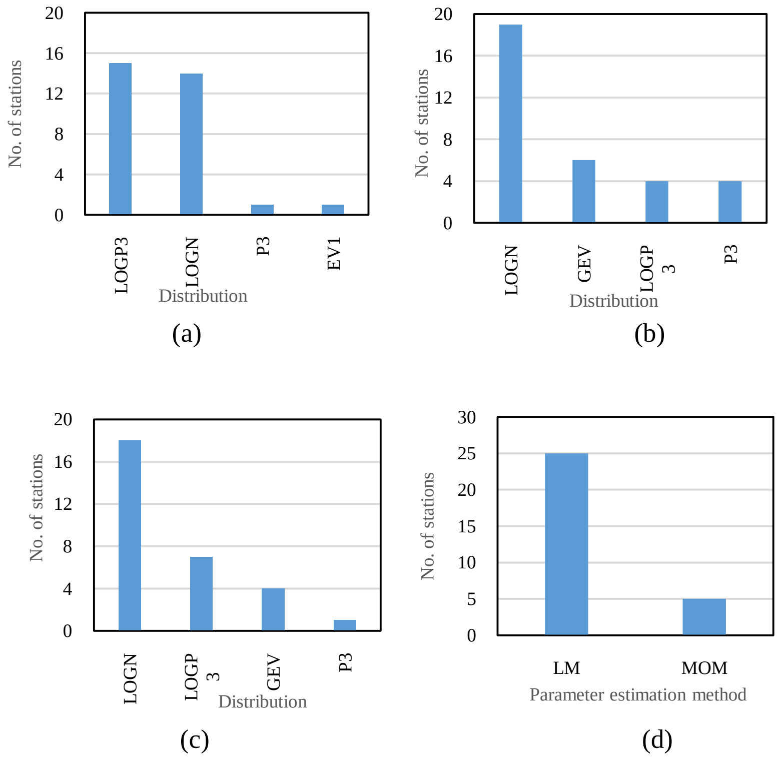
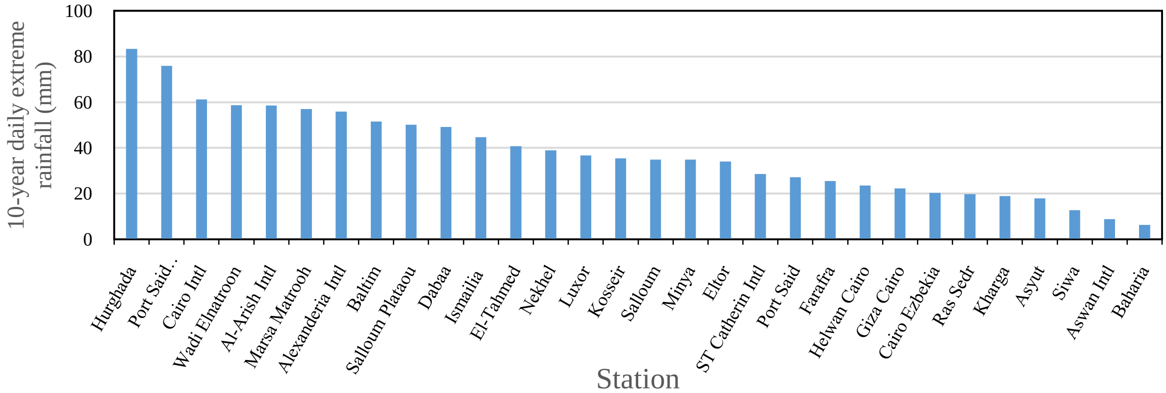


Figure 3. The number of stations for each selected best distribution using: (a) the MOM; (b) the L-moments; (c) the MOM and the L-moments; and (d) the number of stations for each selected best parameter estimation method

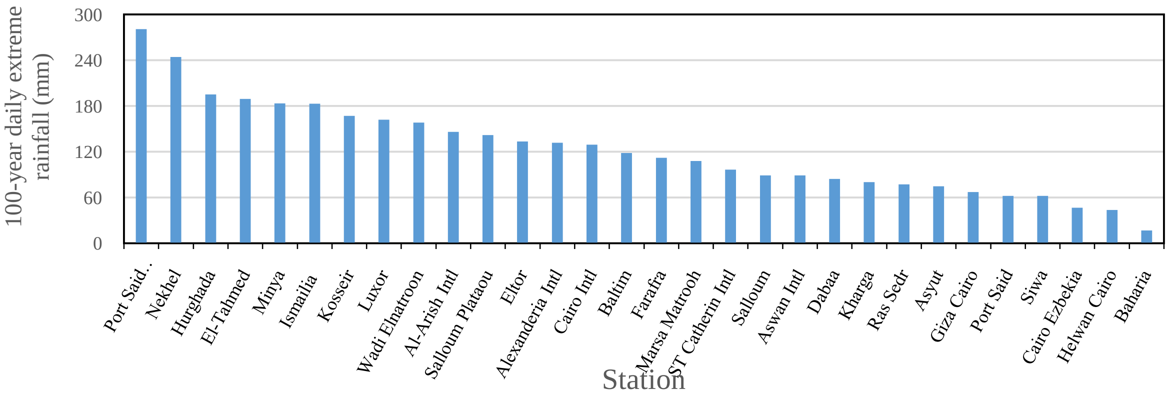
Table 5. The optimum model and Quantile estimates for different return periods for each station.

Station	**Optimum Model**	$P_T(mm)$							
		P_2	P_5	P_{10}	P_{25}	P_{50}	P_{100}	P_{200}	P_{1000}
Al-Arish Intl	LOGN,LM	19.07	39.83	58.52	88.20	114.97	145.98	181.50	284.54
Alexanderia Intl	GEV,LM	24.98	41.32	55.84	79.89	103.03	131.69	167.29	287.14
Aswan Intl	LOGPIII,LM	1.50	4.26	8.81	22.33	44.64	89.01	176.89	876.19
Asyut	LOGN,LM	3.10	9.81	17.90	34.00	51.45	74.74	105.05	212.17
Baharia	LOGN,LM	1.89	4.17	6.30	9.79	13.01	16.82	21.25	34.45
Baltim	LOGN,LM	18.58	36.32	51.56	74.90	95.33	118.48	144.46	217.51
Cairo Ezbekia	LOGN,LM	7.33	14.30	20.27	29.42	37.42	46.48	56.64	85.18
Cairo Intl	PIII,LM	13.79	41.03	61.25	88.06	108.55	129.30	150.23	199.86
Dabaa	LOGN,MOM	20.93	37.92	49.17	63.38	73.92	84.39	94.81	118.97
El-Tahmed	LOGPIII,LM	9.19	23.30	40.70	78.07	122.87	189.12	285.72	710.91
Eltor	LOGN,LM	6.35	19.11	33.98	62.76	93.29	133.35	184.70	361.85
Farafra	LOGN,LM	4.17	13.71	25.54	49.57	76.09	111.94	159.20	329.33
Giza Cairo	LOGPIII,LM	6.59	14.38	22.20	35.99	49.74	67.09	88.73	161.20
Helwan Cairo	LOGPIII,MOM	9.33	17.45	23.46	31.46	37.57	43.75	49.95	64.48
Hurghada	GEV,LM	25.17	57.51	83.38	122.22	156.24	195.16	239.79	370.82
Ismailia	LOGN,LM	7.90	24.65	44.65	84.12	126.65	183.11	256.31	513.15
Kharga	LOGN,LM	3.22	10.29	18.90	36.10	54.85	79.96	112.75	229.23
Kosseir	LOGN,LM	5.28	18.42	35.40	71.01	111.32	166.94	241.57	518.01
Luxor	LOGN,LM	5.95	19.66	36.73	71.49	109.93	161.98	230.69	478.66
MarsaMatrooh	LOGPIII,MOM	20.82	41.53	56.96	77.27	92.53	107.68	122.58	156.25

Minya	LOGN,LM	4.54	17.31	34.83	73.37	118.74	183.22	272.14	615.82
Nekhel	LOGPIII,LM	7.12	20.35	38.89	84.05	144.84	244.28	404.51	1249.03
Port Said	GEV,LM	10.60	19.67	27.21	38.94	49.58	62.10	76.88	122.47
Port Said Elgamil	LOGN,LM	15.26	43.78	75.94	136.58	199.56	280.85	383.54	729.81
RasSedr	LOGN,LM	3.70	11.11	19.73	36.40	54.06	77.21	106.86	209.05
Salloum	LOGN,LM	11.03	23.48	34.85	53.09	69.68	89.02	111.30	176.53
SalloumPlataou	LOGN,LM	13.97	32.32	50.10	79.94	108.11	141.89	181.84	303.44
Siwa	GEV,LM	2.88	7.34	12.69	24.40	39.08	61.93	97.55	277.03
ST Catherin Intl	LOGPIII,MOM	6.90	17.40	28.59	49.02	69.81	96.36	129.68	241.63
WadiElnatroon	LOGN,MOM	17.33	38.59	58.63	91.58	122.16	158.37	200.68	327.15



(a)



(b)

Figure 4. The daily extreme rainfall of the selected stations in Egypt for return periods: (a) 10-year , (b) 100-year

4 CONCLUSIONS

The performance of some of most popular probability distributions was assessed in order to identify the most suitable model that could accurately estimate extreme rainfall events in different regions in Egypt. Annual maximum series were extracted from historical daily rainfall records of 30 stations throughout the country. Six distributions (N, LN, PIII, LPIII, EV1, and GEV) along with two parameter estimation methods (MOM and L-moments) were evaluated by using graphical and numerical criteria. The results indicated that the Log-Normal and Log-Pearson Type III distributions were the best model for most studied stations in Egypt; and the L-moments method was better than the MOM for parameter estimation at the greatest number of stations. Furthermore, the Log-Normal distribution along with the L-moments was the best model for describing extreme rainfall events in Egypt.

The conclusions made in this study represent a starting point with respect to the extreme rainfall estimation in Egypt. Hence, a recommendation for future studies is directly related to the use of more distributions and parameter estimation methods, which is currently being studied by the authors. As the case study consisted of daily rainfall data from only 30 stations, thus, it is suggested to include more data to have more accurate extreme rainfall predictions in whole country.

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