# *IDEAL VELOCITY AS A NEW CONCEPT FOR FLOWS*

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## ABSTRACT

As it is well-known, both fresh water and precipitation have no homogeneous mixture temporally or spatially on the earth planet. Transportation of water from one region to another has existed since the history of humanity. The transfer is achieved by water conveyance structures (i.e. channels, pipes, tunnels, galleries, and drains). However, the flow in such structures occurs hydraulically and can be divided into two main classes, namely, pipe flow and open channel flow. There are many published studies in the current literature performed on modeling pipe or open canal flows. In this study, following the relevant discussion on the existing literature a new concept is introduced as “ideal velocity”. Furthermore, ideal discharge, Reynolds and Froude numbers those are currently used in the computational hydraulics are re-modified according to the new concept. It is considered that the simplification of such equations by the new concept will bring a great convenience in solving hydraulic problems, particularly in both pipe and open canal manufacturing sectors.

**Keywords:** Ideal velocity, pipe flows, pipe hydraulics, open canal flows, pipe manufacturing

## INDRODUCTION

It is well known that the precipitation and fresh water resources are not homogeneously distributed spatially or temporally. For example, one-fourth of annual precipitation occurs at the regions, where one-third of total world population lives. In another word, two-third of world population use only one-fourth of the total fresh water resources (Aytek and Toprak, 2001). Therefore, transfer of water from one region to another (water transport) has existed along with the existence of humanity. In the last few decades, world fresh water consumption has increased unprecedentedly as a result of world population growth, industrialization, technological developments, and diversity in water use based on the rise in living standards. The world population dramatically increases. According to the 2006 Revision (UN), the world population is likely to increase by 2.5 billion over the next 43 years, passing from the current 6.7 billion to 9.2 billion in 2050 (Toprak et al., 2013a). The more population is bound to need more water uses. The more water uses imply more water distribution networks. It is expected that the distribution in the precipitation as well as in fresh water resources will completely change both spatially and temporally as a result of the global climate change (Toprak et al., 2013a and 2013b). This situation means that the fresh water requirement will increase year by year. Again, it is possible to say that the more water demands more water supplies, and therefore, more water distribution networks.

It is clear that for supplying the domestic, industrial, and irrigation water, transfer of water from one region to another (water transport) will be a vital requirement for every time.
The use of fossil energy sources produces the greenhouse gases, and consequently, IPCC reports claim that emission of such gases depends mostly on human activities at high percentages, 98% or 99% (IPCC, 2007; IPCC, 2013-AR5 2014). Among all the publications about 90% of the scientific articles put forward evidences on the existence of the global warming or global climate change (Şen, 2005; Şen, 2009). Alashan et al., (2015) indicated that it is very important

to produce the energy but it is more important to produce clean, sustainable, and inexhaustible energy sources with marginal pollution effects that are almost negligible in practice (i.e., hydropower, solar, wind, wave, geothermal, and hydrogen energy etc.). According to Toprak et al. (2014), hydropower is a key strategy to protect fresh water resources (i.e. rivers) against chemical, biological, nuclear, and physical pollutants. However, water transport is also needed for hydropower.

On the other hand, the global or local water trade and inter-basin water transfer are another two important reasons those effect on water transport.

With the above mentioned consideration it is possible to say briefly that due to inhomogeneous distribution of precipitation and water resources, increase in population, industrialization, technological developments, diversity in water use, global climate change, supplying the domestic, industrial, irrigation, and hydropower water, water trade, and inter-basin water transfer, currently the water transport is globally becoming one of the most important issues and it expected to be a vital requirement for a long time period in the future.

The water transfer is technically made by hydraulics structures i.e. canal, pipe, tunnel, gallery, drain, and balloons. However, the type of flow occurs in those structures are categorized hydraulically as open canal and pipe flows. Due to the vital importance of the issue, there are many published studies performed on the pipe and open canal flow modeling. Some of the valuable published studies are discussed under a separate title.

## A BRIEF DISCUSSION ON THE CURRENT LITERATURE

Toprak (2009) states that to minimize the environmental impacts and maximize the benefits of water resources, optimization of effective section for any water flow is particularly important for irrigation and drainage systems as well as for flood flow in natural channels. The author indicates that up to the end of the 20th century, modelers used well-known empirical conventional equations such as Chezy, Gauckler-Strickler, and Manning approaches for the purpose. However, recently the availability of large capacity and transaction speed of the computers allow the use of the artificial intelligence methods (i.e. fuzzy logic-FL, genetic algorithms-GAs, and artificial neural networks-ANNs).

The relationship between the depth and the discharge is analytically analyzed by Ermolin (2000). An interesting application of the Manning and Strickler formula is presented by Dubos (1988). The impact of the submerged aquatic weeds on the hydraulic performance of open channels is discussed by Abdeen (2006). An optimal design of an open channel section under critical flow condition is presented by Bhattacharjya (2006). Manning's equation is used to specify the uniform flow condition in the channel. The result of a work performed for optimal design of parabolic-bottomed triangle canals is reported by Babaeyan et al. (2000). Another similar two works performed by Das (2000a) and Das (2007a). Das (2007a) indicates that the cost of open channels can be minimized by using (a) the optimal design concept, (b) a new geometric shape to substitute the trapezoidal channels, (c) a composite channel. The Lagrange multiplier technique is used to solve the resulting channel optimization models in the study. Das (2007b) introduced the flooding probability constrained optimum channel design concept to design an optimum channel cross-section for safety against overtopping. A genetic algorithm based optimization model is initially developed by Bhattacharjya and Satish (2007) to determine the factor of safety of a channel slope for given soil parameters. Jain et al. (2004) proposes a non-linear optimization program (NLOP), which is based on a distributed approach that is equivalent to Lotter's observations, which allow spatial variations in velocity across a composite channel cross-section. Monadjemi (1994) performed another work to optimally determine any hydraulic channel section by using Lagrange's method. Chahar (2005) proposes a set of equations each for different cases to optimally design the parabolic canal section. Swamee et. al., (2000) has comprehensively designed the minimum cost for any irrigation canal section.

Siam (2002) presented a model for the controlled flow of a fluid through a network of canals using a coupled system of St. Venant equations.

Except from the above discussed studies, there are many published works in the literature. However, due to page limitation the rest are given in chronological order as Meniconi et al. (2014), Wertel et al. (2010), Froehlich (2008), Kentel and Aral (2007), Chahar (2005), Depeweg and Urquieta (2004), Siam (2002), Swamee et al. (2002a), Swamee et al. (2002b), Swamee et al. (2001), Das (2000b), Swamee et al. (2000a), Swamee et al. (2000b), Swamee et al. (2000c), Hankin and Beven (1998), Federico (1998), Reddy (1996), Reddy (1995), Swamee (1995), Froehlich (1994), Garcianavarro et al. (1994), Loganathan (1991), Flynn ve Marino (1987), Guo and Hughes (1984), Mironenko et al. (1984).

The above mentioned valuable studies offer some modification of existing equations or propose several modeling techniques for optimization of artificial or natural channels' cross-section. The value of these studies is due to the fact that the researchers try to investigate the possibility of getting a maximum flow with a minimum energy or minimum construction cost in a canal or a pipe. However, most of the analyses made in these works and their results are not simple, oppositely the problem is solved with several very complex differential equations those require commercial software. On the other hand, no one talk about a novel approach or a new perspective on the flow in an open canal or in a pipe. In this study, a new concept as "ideal velocity" is introduced. Furthermore, the well-known equations used in computational hydraulic are re-modified according to the new concept. The "ideal velocity" depends on the assumption of circular distribution of the velocity through the cross-section of pipe or an open canal flow. In engineering studies, it is too difficult to model any natural events with deterministic and analytical models due to the complexity of mathematical expressions. It is considered that the simplification of such equations by using the new concept will bring a great convenience in solving hydraulic problems, particularly in pipe and open canal manufacturing sectors.

## IDEAL VELICITY CONCEPT

Ideal velocity concept is introduced in this study that does not exist in the current literature. Its definition is inspired by the "ideal gas" and "ideal fluid" concepts. Sometimes, although any event is well read and well interpreted, however every time it is not possible to reflect exactly the same reading or interpretation mathematically in the model. Just as Einstein (1879 - 1955) stated, "So far as the laws of mathematics refer to reality, they are not certain and so far as they are certain, they do not refer to reality. This difficulty can be explained by the well-known butterfly effect theory of Edward Norton Lorenz (1917 – 2008) who is the father of chaos theory or the theory of Zadeh (1921 – ?) as "the closer one looks at a real-world problem, the fuzzier becomes its solution" and as Henri Poincaré (1854 – 1012) indicated that "very simple systems maybe exhibit a highly complex dynamic". Şen (2004) states that all the scientific methodologies such as analytical, probabilistic, statistical, stochastic dynamical modeling techniques require two things: 1) a model with a set of assumptions, and 2) data for the verifications. The assumptions are for idealization of the phenomenon. Hence, the assumptions introduce a sort of filtering effect, which may lose the imprecise parts of the basic information. Therefore, it is thought that the assumption made in deriving the "ideal velocity" concept become convenient for such situations. Figure 1 shows the possibility of alternative models of any natural event. In the figure, the first square shows a real natural event which is in white color, which represents the accuracy of the event. This means that the square is white so the event is accurate. Model 1 represents the model of the nature, which has only four independent variables. Model 2 is another model of the same event with 16 independent variables. As it can be seen from these two models, their errors which are shown by the shaded area are equal to each other. Model 3 has less shaded area than Model 2. In other word, it has less error and its accuracy is higher than the first two models. Although the natural event is modeled by less number of variables comparatively to the Model 2, its accuracy (increasing the white area) is higher because the impact of independent variables has been taken into account with different coefficients.

From the above made explanations, it is possible to say that there is a complexity and chaos in natural events, so it is convenient to idealize or approximately model to any natural event as made in this study in definition of “ideal velocity”.

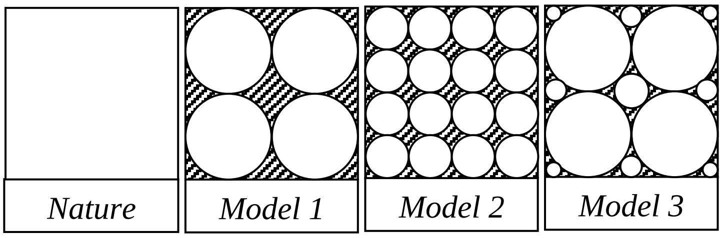

**Figure 1 A natural event and possible alternative three models**

## DERIVATION OF IDEAL VELOCITY IN PIPES FLOWS

Herein, the cross-sectional flow velocity, which is parabolically distributed through the section, is assumed to be circular. Ideal velocity, which is proposed as a new concept, is defined as circular cross-sectional velocity. In other words, the parabolic cross-sectional velocity is idealized. According to ideal velocity definition, the velocity can be expressed by diameter (*D*, *m*) for the pipe flows.

By assumption of ideal velocity distribution, the ideal mean cross-sectional flow velocity ($V_i$, *m/s*) of a pipe, which has a diameter of *D* (as given in Figure 2) can be derived as follows.

$$V_i = \frac{\dfrac{\left\lfloor \dfrac{\pi D^2}{4} \right\rfloor}{2}}{D} = \frac{\pi D}{8} \cong 0{,}393\, D \tag{1}$$

In this case, the maximum cross-sectional velocity ($u_{\max}$) will be equal to semi diameter of the pipe. Equation (1) is not dimensionally homogeneous as well-known Manning-Strickler equation.

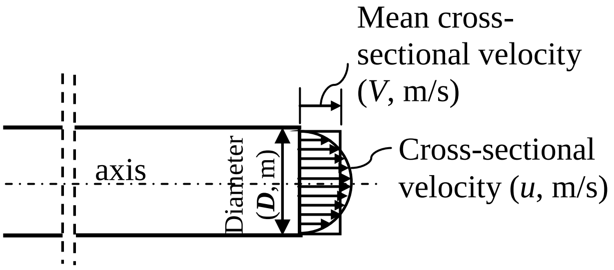

**Figure 2 The distribution of the cross-sectional velocity in pipe flows**

## DERIVATION OF IDEAL VELOCITY IN OPEN CANAL FLOW

In the derivation of ideal velocity for open canal flows the analyses made for the pipe flows is used. Again the cross-sectional flow velocity is assumed as circular. According to ideal velocity definition, the velocity can be expressed by depth (*y*, *m*) for open canal flows.

By assumption of ideal velocity distribution, the ideal mean cross-sectional flow velocity ($V_i$, *m/s*) of an open canal, which has a depth of *y* (as given in Figure 3) can be derived as following. In this case, maximum cross-sectional velocity ($u_{\max}$) will be equal to the depth of the flow.

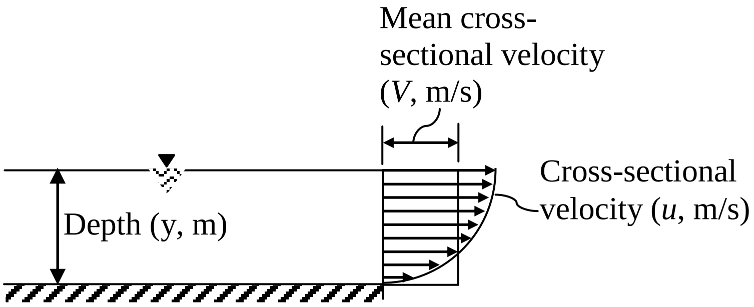

**Figure 3 The distribution of the cross-sectional velocity in open canal flows**

$$V=\frac{\frac{\left\lfloor \frac{\pi D^2}{4} \right\rfloor}{4}}{y} \qquad (2)$$

From Figure 3, it can be assumed that the depth is equal to semi diameter of the velocity distribution as given in Eq. (3)

$$y=\frac{D}{2} \Rightarrow D=2y \qquad (3)$$

Accordingly, the mean cross-sectional velocity can be obtained as presented in the following expression.

$$V_i=\frac{\frac{\left\lfloor \frac{\pi * 4y^2}{4} \right\rfloor}{4}}{y}=\frac{\pi * y}{4}=0{,}785y \qquad (4)$$

This equation is also non-dimensionally homogeneous as stated previously.

## SEVERAL APPLICATIONS OF IDEAL VELOCITY

### 1. Ideal Discharge

By substituting the ideal velocity into continuity equation, the ideal discharge will be obtained in a form as presented in Eq. (9). The derivation steps are given in Eqs. (5)-(8).

$$Q=\int_A udA=V*A \qquad (5)$$

$$V=\frac{\int_A udA}{A}=\frac{Q}{A} \qquad (6)$$

$$\frac{\pi D}{8}=\frac{Q}{A} \qquad (7)$$

$$\frac{\pi D}{8}=\frac{Q}{\frac{\pi D^2}{4}} \qquad (8)$$

$$Q_i=\frac{\pi^2 D^3}{32}=0{,}3084D^3 \qquad (9)$$

Herein, *Q*, *Q<sub>i</sub>*, and *A* are the discharge, ideal discharge, and cross-section area, respectively. Eq. (9) is dimensionally not homogenous.

## 2. Ideal Reynolds Number

By substituting the ideal velocity into the equation of Reynolds Number (Eq. 10), the ideal Reynolds Number will be obtained in a form as presented in Eq. (11).

$$R_e = \frac{VL}{\nu} \tag{10}$$

and hence,

$$R_{ei} = \frac{\frac{\pi D}{8} D}{\nu} = \frac{\pi}{8\nu} D^2 = 0.393 \frac{1}{\nu} D^2 \tag{11}$$

Herein, $R_e$, $R_{ei}$ and $\nu$ are Reynolds Number, ideal Reynolds Number, and cinematic viscosity. $R_{ei}$ has a coefficient of 0.393 that should have a dimension of $\frac{1}{s^2}$ in both SI and MKS unit systems.

### 3. Ideal Froude Number

By substituting the ideal velocity into the equation of Froude Number (Eq. 12), the ideal Froude Number will be obtained in a form as presented in Eq. (13).

$$Fr = \frac{V}{\sqrt{gy}} \tag{12}$$

and therefore,

$$F_{ri} = \frac{\frac{\pi y}{4}}{g^{1/2} y^{1/2}} = \frac{\pi y}{4 g^{1/2} y^{1/2}} = \frac{\pi y^{1/2}}{4 g^{1/2}} = 0{,}251 y^{1/2} \tag{13}$$

Herein $F_r$, $F_{ri}$, and $g$ are Froude Number, ideal Froude Number, and acceleration of gravity. The $F_{ri}$ is dimensionless; therefore the coefficient of 0.251 should have a dimension of $\mathrm{m}^{-1/2}$ in both SI and MKS unit systems

## CONCLUSION

In this study, a new concept namely "ideal velocity" is introduced and it can be defined as the assumption that a cross-sectional velocity to be circular in both pipe and open canal flows. Therefore, it can be expressed approximately as $0.393D$ for the pipe flows and $0{,}785y$ for open canal flows, where $D$ and $y$ are the diameter and depth variables, respectively. Ideal discharge can be defined as $0.308D^3$ for the pipe flows. The expression of the ideal discharge varies for the open canal flow according to the geometry of the canal (trapezoidal, triangular, rectangular, or square). Ideal Reynolds and ideal Froude Numbers can be expressed in terms of the diameter for pipe flows and the depth for open canal flows as explained in the text of this paper. It is considered that the simplification of such equations by using the new concept will bring a great convenience in solving the hydraulic problems, particularly in both pipe and open canal manufacturing sectors.

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