EVALUATION AND MODIFICATION OF SOME EQUATIONS USED IN DESIGN OF SUBSURFACE DRAINAGE SYSTEMS

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ABSTRACT

This research evaluates some unsteady state equations that are used in designing subsurface drainage systems and conducts some modifications to match the conditions in Delta Region.Three unsteady state equations that selected for this study are: Glover and Dumm – Hamad - Luthien.

Mashtool Pilot Area (MPA) in Delta Region was selected to be the case study of this research. To make the evaluation, hydraulic head above drain (h_t) was collected for three wells (h1, h2, and h3) located in Unit (1) at MPA. These three wells had been constructed in 1980. These data were during year 1987 (i.e. seven years only after construction) to be ideal enough for evaluating the equations and making some modifications in these equations to fit theconditions in Egypt. The ideal values for drawdowncurves that were conducted by previous studies were used to make the validation of these modifications.

Keywords: Subsurface Drainage Systems, Mashtul Pilot Area, Unsteady State Equations,Glover and Dummy, Hamad, Luthien

1. INTRODUCTION

Land drainage is the removal of excess surface and subsurface water including the removable of soluble salts from root zone, to enhance soil aeration and soil temperature and as a result enhance crop growth and maintain the soil productivity **(ICID 1978).** A surface drainage system is applied when the waterlogging occurs on the soil surface, whereas a subsurface drainage system is applied when the waterlogging occurs in the soil **Abd-eldayem, 1998**.

Nijland et al., 2005mentioned that theadvantages of subsurface drainage system are: (1) it can effectively solve the drainage problems at relatively moderate costs with a minimum interference to agricultural practices and existing infrastructure, (2) its maintenance is less problematical than maintenance of other forms of drainage, and (3) it increases land area used for agriculture (it saves about 5% of land). Other important advantagethat can be added is using subsurface drainage systems conserves water from contamination so that it can be reused safely and protect environment from negative impact caused by using surface drainage systems.

For the above advantages, Egypt is implementing subsurface drainage systems in large areas of agricultural lands. The Egyptian Public Authority for Drainage Projects (EPADP) is not only heavily engaged in the construction of new drainage systems but has also become responsible for the drainage system management and rehabilitation for a large area. Consequently, (EPADP) is interested in the economic viability of establishing these drainage systems **(Abdel-Dayem, 1973).**

Mashtul Pilot Area (MPA), which has been supplied with subsurface drainage system since 1980 in southeastern part of the Nile Delta, was selected to be the case study of this research as it is an old area with alluvial soil (**Rashad, 2003)** and it can be considered a representative area for Delta Region.

Determining space between laterals is conducted by using two theories: **(i)** the steady state theory that based on the assumption that the rate of recharge to the groundwater is uniform and steady and that it equals to the discharge through the drainage system. Thus, the water table remains at the same height as long as the recharge continues, **(ii)**the un-steady state theory, whichbased on, the movement of water table through the soil is a transient condition and the hydraulic head at any point in the soil is not a constant and changes with time.The un-steady state concept is more realistic than the steady state one. Consequently, this research selected three unsteady state equations to be evaluated and modified to reach more accurate solutions according to conditions in Egypt.

Three unsteady state equations,whichare selected for this study,are Glover and Dumm – Hamad - Luthien. Each equation has its own assumptions and has its own variables.
Microsoft excel was used for fitting data with the equations for making some modifications in the constants of these equations. These modifications were done to fit data collected from three wells (h1, h2, and h3) located in Unit (1) at MPAforyear 1987. Validation of these modifications were conducted to fit ideal values of $h_t$according to **Eissa et al. 1996 & Eissa, 2001that are illustrated in table (1).**

The average values to describe the soil characteristics at Unit (1) at MPA were as follow:

Soil hydraulic conductivity (k) = 0.027 m/d

The corresponding value of drainable porosity (*f*) = 0.011 (calculated by using Subsurface Drainage Design Program(SDDP) compensating with values of the ideal irrigation cycles for drain depth equales1.2 m**(Rashad, 2003).**

The distance between the axe of laterals and the impermeable layer (D) = 5 m.

The following parameters are the design criteria of the laterals in the area:

The distance between laterals (L) = 15 m
The radius of laterals (r_o) = 0.04 m
Depth of drains from ground surface = 1.2 m
Hydraulic head (after six days) = 0.3 m
The equivalent depth (d_e) can be taken = 1.5 m (by compensation in Glover-Dumm's equation with the previous data)

2. METHODOLOGY

For achieving the objectives of this research, the following research tasks have been undertaken:

1- Three unsteady state equations were selected to be evaluated(Glover and Dumm – Hamad – Luthien)
2- Data for three drains during 1987 (seven years after construction) were collected to be ideal enough for evaluating these equations and making some modifications.

3- Some models by excel program were created to fit the original equations withthe data collectedthen modifications in some constants of these equations were proposed.
4- Data of ideal irrigation cycles in Delta region, which have been concluded in previous studies, were used for validation of the modificationsthathad been conducted.

3. GLOVER DUMM'S EQUATION

Under unsteady state condition **Dumm, 1954** introduced a formula proposed by Glover. This formula is based on the solution of heat flow equation. In his solution Dumm considered the initial water table as a forth degree parabola instead of the assumption of a flat one.**Dumm, 1954** used the differential to describe the fall of the water table. His solution, which is based on a formula developed by Glover, describes the lowering of an initially horizontal watertable as a function of time, place, drain spacing, and soil prosperities.
Assumptions of Glover and Dum equation are: irrigation is inregular intervals, soil is uniform, and movement of water in soil is directly proportional to hydraulic gradient.
The final equation given by Glover-Dumm can be written as follow:

$$L^2 = \pi^2 \left(\frac{K d_e t}{f} \right) \left(\ln \frac{4h_o}{\pi h_t} \right)^{-1}$$

Where, h_t = height of watertable midway between laterals at time t (m);h_o = initial height of the watertable midway between laterals;k = hydraulic conductivity (m/day);d_e = equivalent depth of the soil layer below drain level (m);*f*= drainable porosity (dimensionless);L = drain spacing (m); and t = time after instantaneous rise of water table(day). **Figure (1)** illustrates these parameters.

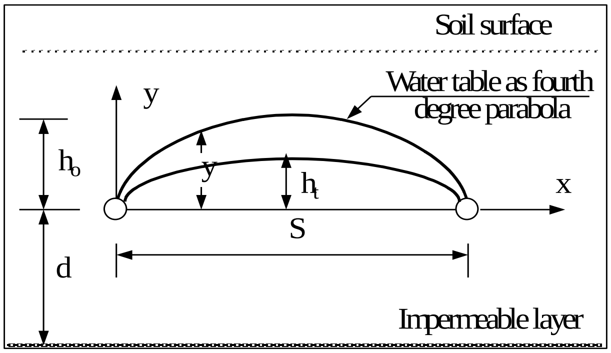


Figure (1) Geometry of drainage system (as assumed by Dumm)

Table (1) Different parameters of the Watertable Draw Down Curve

Days after Irrigation	Hydraulic head above drain level (m) (for drain depth 1.2 m &ideal Hyd. Head 0.3m)			Hydraulic head above drain level (m) (for drain depth 1.4 m &ideal Hyd. Head 0.4m)			Hydraulic head above drain level (m) (for drain depth 1.7m &ideal Hyd. Head 0.8m)		
	Average	Upper boundary	lower boundary	average	Upper boundary	lower boundary	average	Upper boundary	lower boundary
1	0.85	1.05	0.63	1.05	1.26	0.87	1.26	1.45	1.11
2	0.69	0.87	0.50	0.88	1.07	0.67	1.15	1.32	1.00
3	0.56	0.73	0.40	0.73	0.91	0.52	1.05	1.21	0.90
4	0.45	0.61	0.32	0.58	0.78	0.40	0.96	1.11	0.81
5	0.37	0.51	0.25	0.48	0.66	0.31	0.88	1.02	0.74
6*	0.30	0.43	0.20	0.39	0.57	0.24	0.80	0.93	0.67
7	0.24	0.36	0.16	0.31	0.48	0.18	0.73	0.86	0.61
8	0.20	0.30	0.13	0.28	0.41	0.14	0.67	0.79	0.56

3.1. Evaluation of Glover-Dumm's equation

The above equation can be written in other form to get (h_t) as a function in (t):

$$h_t = h_o\left(\frac{4}{\pi}\right)(e)^{-\left(\frac{k d_e \pi^2 t}{f L^2}\right)}$$

This equation was used for fitting data of year 1987 and getting the accuracy of it. To conduct this study, it was necessary to make some changes in one of theconstants.Thisconstantwasselected to bethe power for non-dimensional item in the equation. As a result, the equation can be written as follow:

$$h_t = h_o\left(\frac{4}{\pi}\right)(e)^{-\left(\frac{k d_e \pi^c\, t}{f L^2}\right)}$$

Where (C)isa constant to be checked to get the accuracy of the equation. The ideal values of draw down curves in year 1987 were used for checking the equation by using the Excel program. The average value of (C) that achieved the minimum error of this fitting was shown in **figure (2).**

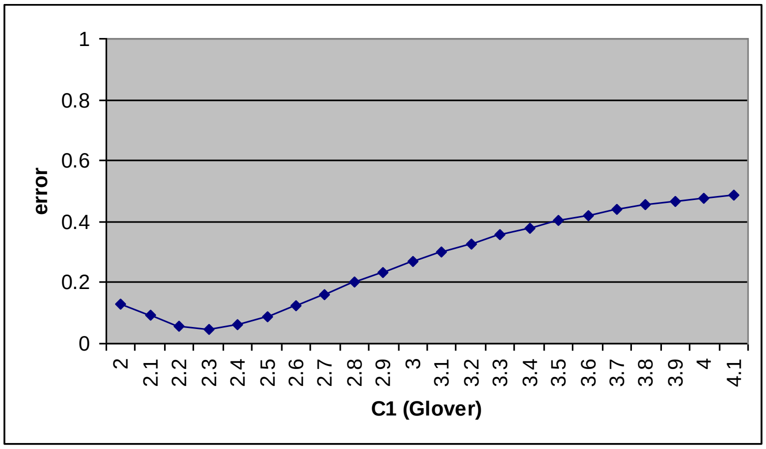
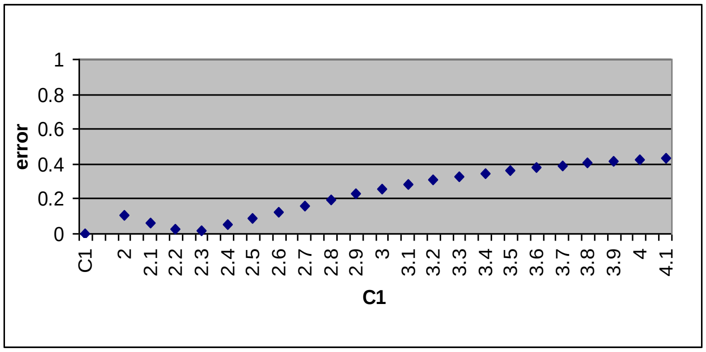


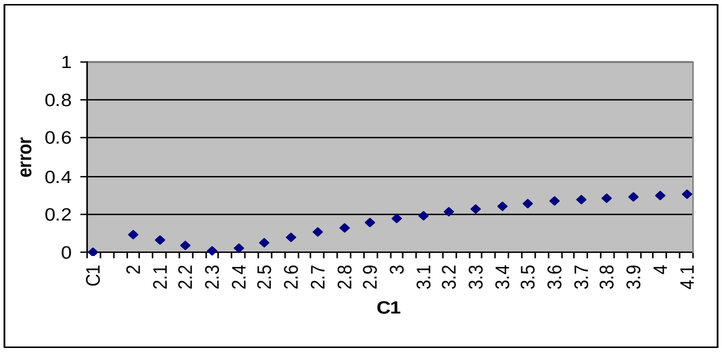
Figure (2) Average value of C in Glover Dumm's equation according to some ideal irrigation cycles in year 1987

3.2. Validation of the results

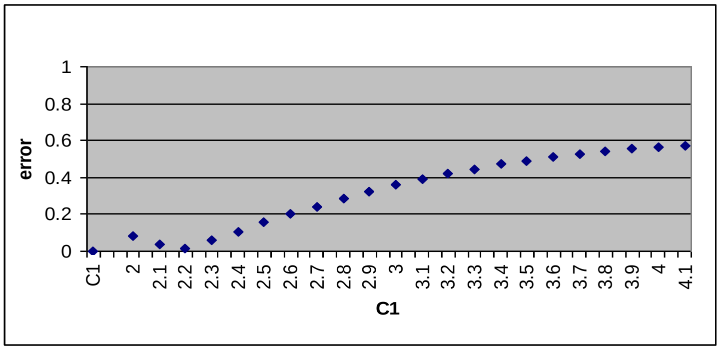
The ideal values of draw down curvesaccording to**table (1)** used to conclude values of (C). **Figures from (3) to (5)**show the different values of (C). The values of (C)that gave the minimum error from these curves are 2.3, 2.3, 2.2, 2.2, 2.4, 2.1, 2.3, 2.4 and 2.3 with average value of 2.28. This result is approximately equal to the result that shown in **figure (2).**



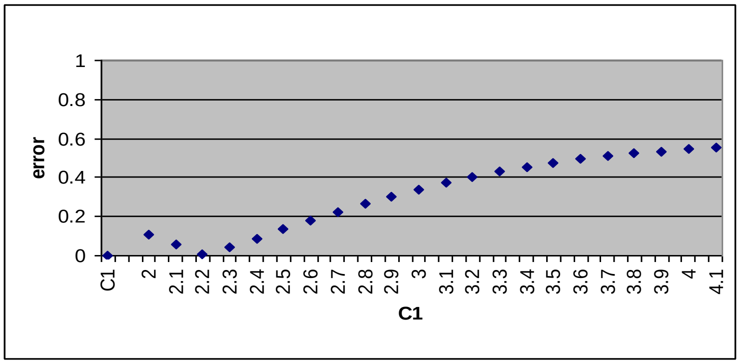
The value of C according to ideal values of DDC



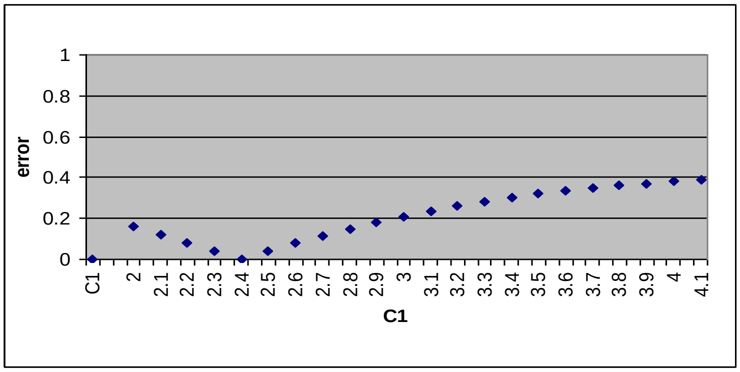
The value of C according to lower boundary values of DDC



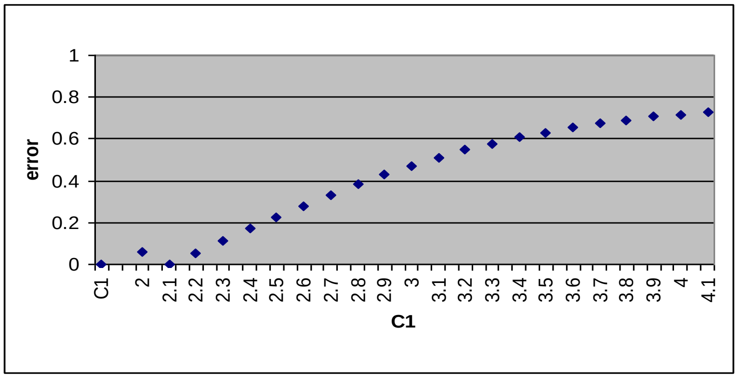
The value of C according to upper boundary values of DDC
Figure (3) The values of Cin Glover Dumm’s equation in case of d.d=1.2m



The value of C according to ideal values of DDC

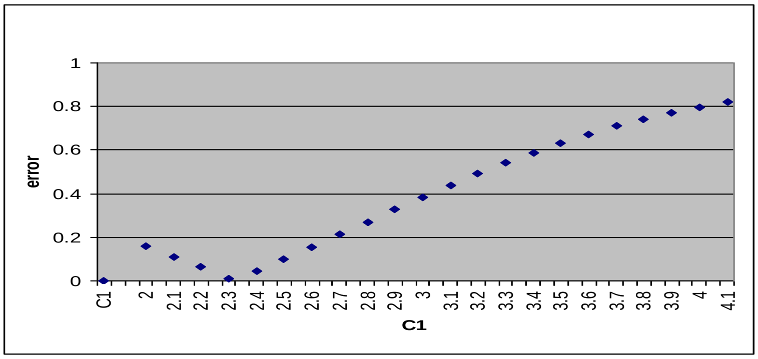


The value of C according to lower boundary values of DDC

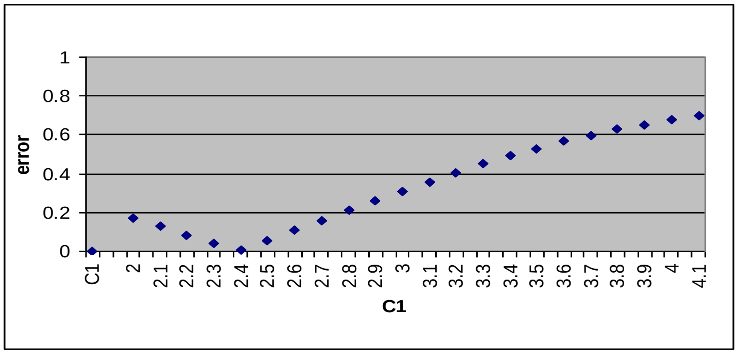


The value of C according to upper boundary values of DDC

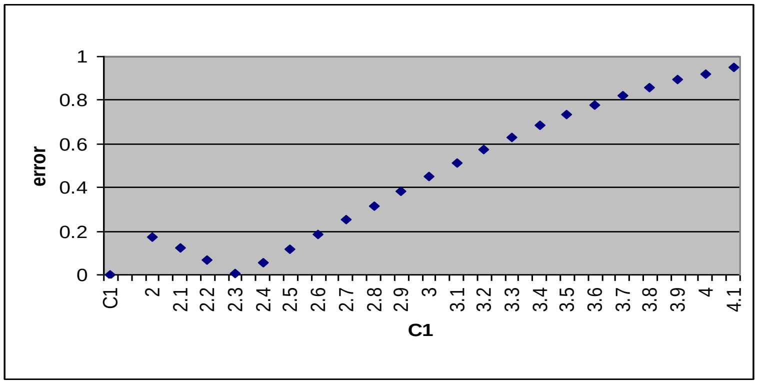
Figure (4) The values of C in Glover Dumm's equation in case of d.d=1.4m



The value of C according to ideal values of DDC



The value of C according to lower boundary values of DDC



The value of C according to upper boundary values of DDC

Figure (5) The values of C in Glover Dumm's equation in case of d.d=1.7m

3.3.The difference in calculating space between laterals by using Glover Dumm's equation

From **section 3.2,**C= 2.28 was chosen for modifying Glover Dumm's equation to fit conditions in Egypt and to avoid over design. As a result, the modified equation can be written as follow:

$$L^2 = \pi^{2.28}\left(\frac{K d_e t}{f}\right)\left(\ln\frac{4h_o}{\pi h_t}\right)^{-1}$$

The space between laterals was calculated by usingboth theoriginal and the modifiedGlover Dumm's equation and recorded in **table (2).**The difference was also calculated by the following equation and recorded in the same table.

$$difference(e) = \left(\frac{L_2 - L_1}{L_2}\right) X100$$

Where,L_1= space between laterals calculated by the original equation**and**L_2= space between laterals calculated by the modified equation

Table (2)Difference incalculating space between lateralsby using Glover Dumm's equation

	d.d =1.2			**d.d =1.4**			**d.d =1.7**		
	L.B	**IDD**	**U.B**	**L.B**	**IDD**	**U.B**	**L.B**	**IDD**	**U.B**
L_1	12.638	13.12	13.9	11.85	13.64	14.6	17.72	18.245	18.5
L_2	14.84	15.4	16.31	13.91	16	17.13	20.8	21.42	21.72
Difference(%)	14.8	14.8	14.8	14.8	14.8	14.8	14.8	14.8	14.8

; LB: Lower Boundary, **IDD:** Ideal Draw Down, **UB:** Upper Boundary

Table (2) illustrates that the difference = 14.8 %and the space between laterals calculated by the original equation is less thanthe space between laterals calculated by the modified equation. Consequently, the original equation results in over design and the modified one is economic for design.

4. LUTHIEN’S EQUATION

Under unsteady state condition **Luthien, 1949** introduced formula for calculating the space between laterals. In his solution,Luthienconsidered the initial watertable as an ellipse instead of the assumption of a flat one.

The final equation given by Luthienwas as follow:

$$L = \left(\frac{4\,C\,K t}{f\ \ln(h_o/h_t)} \right)$$

Where, h_t = height of the water table midway between laterals at time t (m);h_o = initial height of the water table midway between laterals at t = 0 (m);k = hydraulic conductivity (m/day); f= drainable porosity (dimensionless); L = drain spacing (m);t = time after instantaneous rise of water table(day) and C = constant depends on soil type (C = 0.1 for k = 0.025 m/day)

4.1. Evaluation of Luthien’sequation

The above equation can be written in other form to get (ht) as a function in (t):

$$h_t = h_o\,(e)^{-(\frac{4c\,k\,t}{f\,L})}$$

Where C is a constant to be checked to get the accuracy of the equation. The ideal values of draw down curves in year 1987 were used for checking the equation by using the Excel program. The average value of C, which achieved the minimum error of this fitting, was shown in **figure (6)**which illustrates that (C) equal to (0.225).

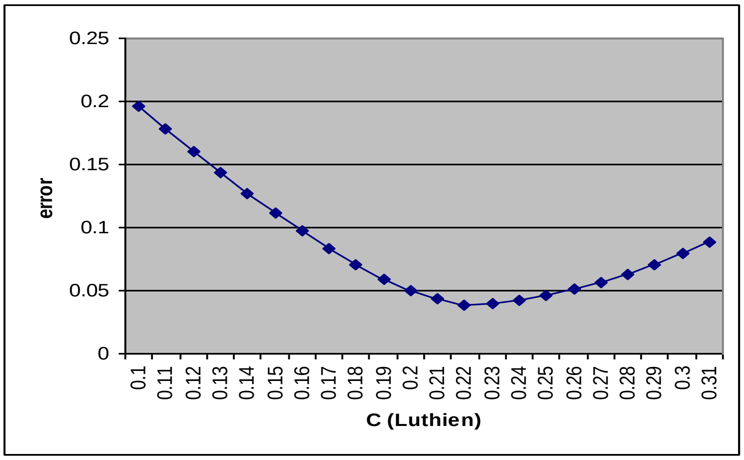
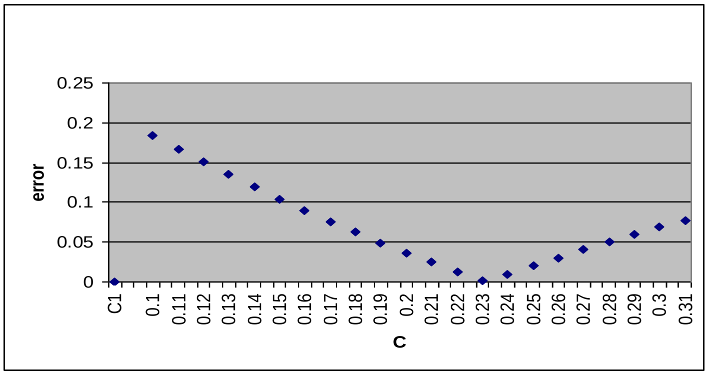


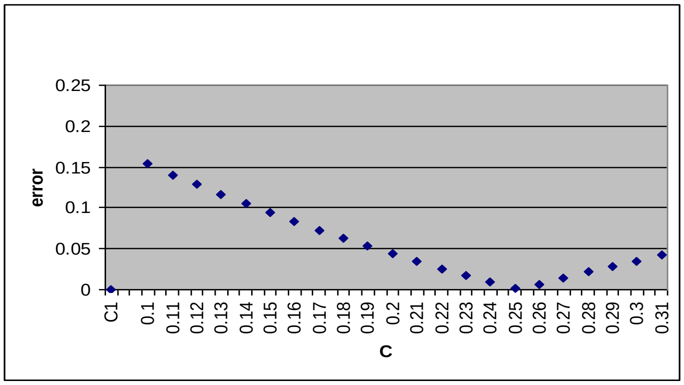
Figure (6) Average value of C in Luthien’s equation according to some ideal irrigation cycles in year 1987

4.2. Validation of the results

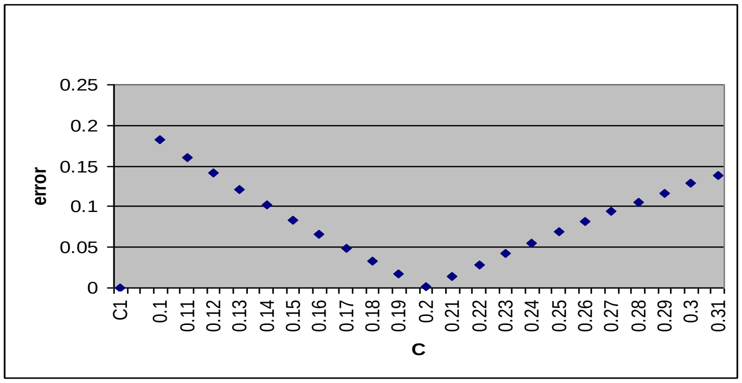
The ideal values of draw down curvesaccording to**table (1) were** used to conclude values of C. **Figures from (7) to (9)**show the different values of C. The values of C, which gave the minimum error from these curves, were 0.23, 0.25, 0.2, 0.215, 0.28, 0.18, 0.19, 0.21 and 0.185 with average value of 0.214. This result is approximately equal to the result that is shown in **figure (6).**



The value of Caccording to ideal values of DDC

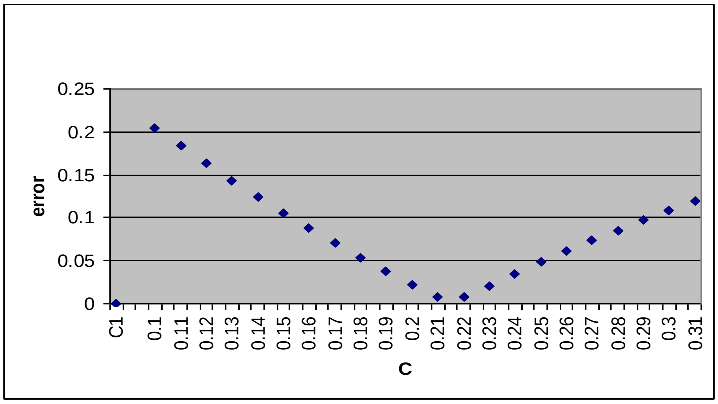


The value of Caccording to lower boundary values of DDC

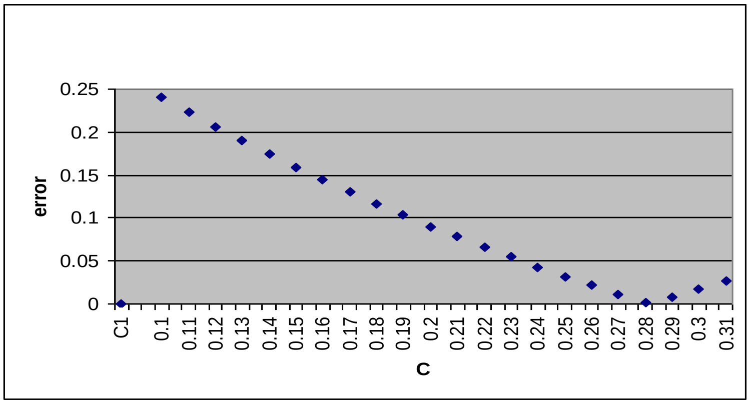


The value of Caccording to upper boundary values of DDC

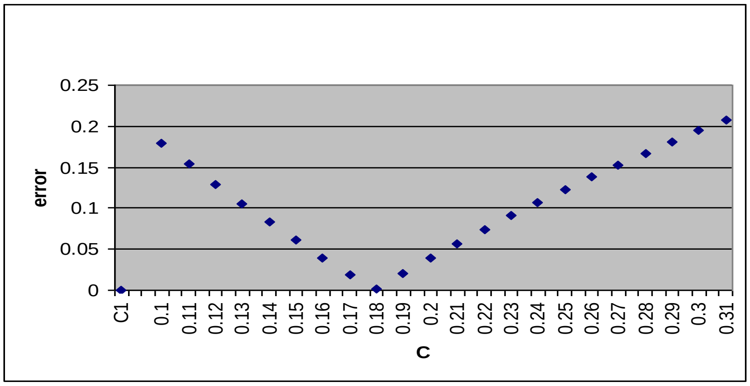
Figure (7) The values of C in Luthien equation in case of d.d=1.2m



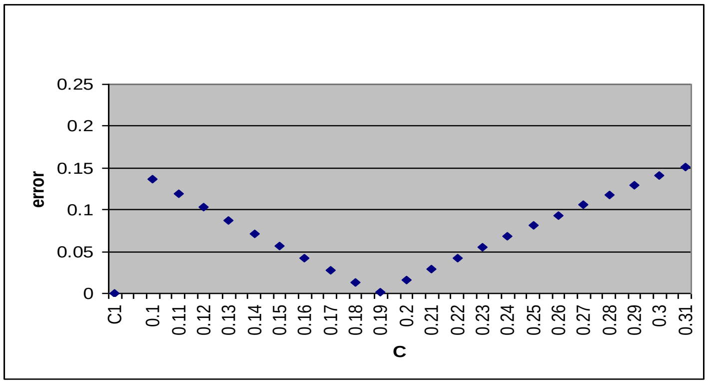
The value of Caccording to ideal values of DDC



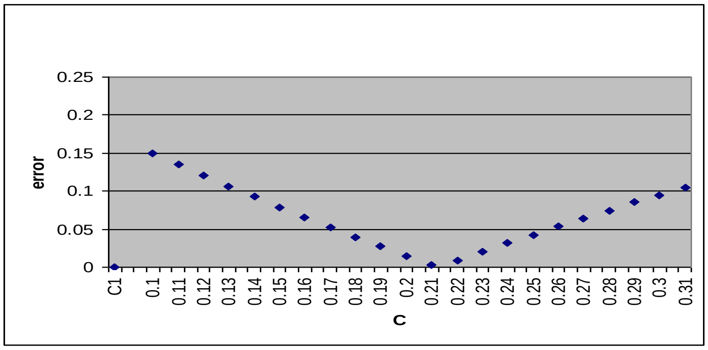
The value of Caccordingto lower boundary values of DDC



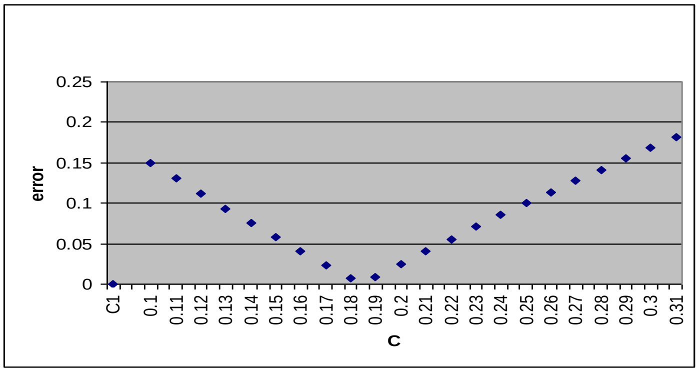
The value of C according to upper boundary values of DDC
Figure (8) The values of C in Luthien equationin case of d.d=1.4m



The value of C according to ideal values of DDC



The value of Caccording to lower boundary values of DDC



The value of C according to upper boundary values of DDC
Figure (9) The values of C in Luthien equation in case of d.d=1.7m

4.3. The difference in calculating space between laterals by using Luthien's equation

From **section 4.2,** C= 0.214. Then, the modified Luthien's equation will be as follow:

$$L=\left(\frac{4\ x0.214Kt}{f\ \ln\left({h_o}/{h_t}\right)}\right)$$

While the original one is:

$$L=\left(\frac{4\ x0.1\ Kt}{f\ \ln\left({h_o}/{h_t}\right)}\right)$$

The space between laterals was calculated by usingboth theoriginal and the modifiedLuthien'sequation and recorded in **table (3).**The difference also calculated by the following equation and recorded in the same table.

$$difference(\%)=\left(\frac{L_2-L_1}{L_2}\right)X100$$

Where, L1= space between laterals calculated by the original equation**and** L_2= space between laterals calculated by the modified equation

Table (3)Difference incalculating space between lateralsby using Luthien equation

	d.d =1.2			d.d =1.4			d.d =1.7		
	L.B	**IDD**	**U.B**	**L.B**	**IDD**	**U.B**	**L.B**	**IDD**	**U.B**
L_1	4.974	5.425	6.266	4.297	5.939	6.992	11.474	12.43	12.926
L_2	10.645	11.61	13.41	9.196	12.71	14.963	24.555	26.6	27.664
Difference(%)	53.27	53.27	53.27	53.27	53.27	53.27	53.27	53.27	53.27

LB: Lower Boundary, **IDD:** Ideal Draw Down, **UB:** Upper Boundary

Table (3) illustrates that the difference = 53.27 % which is a very high percentage and the space between laterals calculated by the original equation is less thanthe space between laterals calculated by the modified equation. Consequently, the original equation results in over design and the modified one is economic for design. It is also very worthy mentioning that the values of the space between laterals calculated by the original equation are very small, costly and unreasonable. It is also obvious that the variation in the value of (C) causes large variation in the value of (L). Then, it is very important to determine this constant for different types of soil in high accuracy.

5. HAMAD’S EQUATION

This equation was chosen in this study to represent the Egyptian contribution in the field of the design of subsurface drainage systems.**Hamad, 1994**mentionedthatunder unsteady state condition, he introduced a formula for calculating the space between laterals in 1962. In his solution he considered the initial watertable as a flat level that tends to decrease at the summit of laterals (i.e. there is hydraulic head at lateral summit). **Hamad** also used the radius of the lateral to be one of the variables in his formula.

The final equation given by **Hamad**was as follow:

$$L = \left(\frac{2\,\pi\,K t}{f\ \ln\left(h_o/h_t \right)\ \ln\left(L/\pi\, r_o \right)} \right)$$

Where, h_t = height of water table midway between laterals at time t (m);h_o = initial height of water table midway between laterals;K = hydraulic conductivity over (m/day);*f*= drainable porosity (dimensionless); L = drain spacing (m);t= time after instantaneous rise of water table(day) and r_o = radius of lateral (m)

5.1. Evaluation of Hamad’sequation

The above equation can be written in other form to get (ht) as a function in (t):

$$h_t = h_o\,(e)^{-\left(\frac{2\,\pi\,k\,t}{f\ L\,\ln\left(L/\pi\, r_o \right)} \right)}$$

This equation was used for fitting data of year 1987 and getting the accuracy of it. To conduct this study, it was necessary to make some changes in one of theconstants.This constant was selected to be as power for a non-dimensional item in the equation. As a result, the equation can be written as follow:

$$h_t = h_o\,(e)^{-\left(\frac{2\,\pi^{c}\,k\,t}{f\ L\,\ln\left(L/\pi\, r_o \right)} \right)}$$

Where C is a constant to be checked to get the accuracy of the equation. The ideal values of draw down curves in year 1987 were used for checking the equation by using Excel program. The average value of C thatachieved the minimum error of this fitting was shown in **figure (10).**This figure illustrates that the constant C = 0.68.

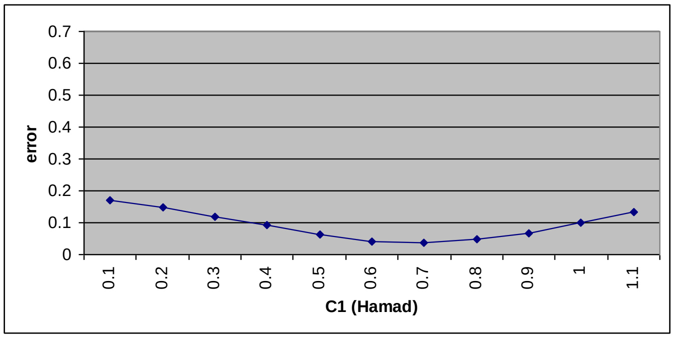
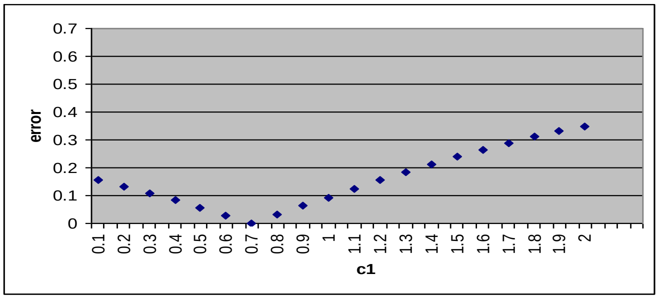


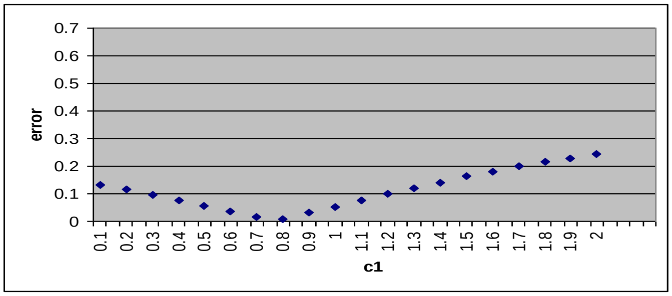
Figure (10) Average value of C in Hamad equation according to some ideal irrigation cycles in year 1987

5.2. Validation of the results

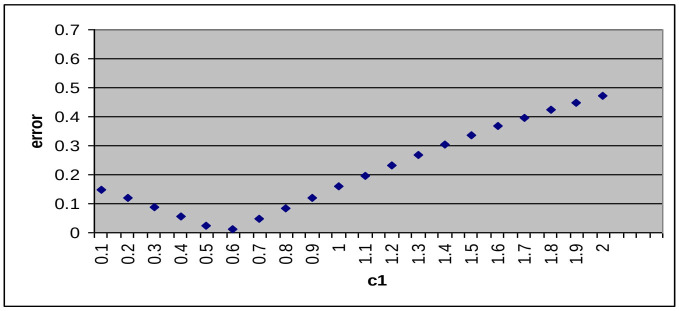
The ideal values of draw down curvesaccording to**table (1) were** used to conclude values of C. **Figures from (11) to (13)**show the different values of C. The values of Cthatgave the minimum error from these curves were 0.7, 0.75, 0.6, 0.6, 0.85, 0.5, 0.5, 0.6 and 0.5 with average value of 0.622.This result is approximately equal to the result thatshown in **figure (10).**



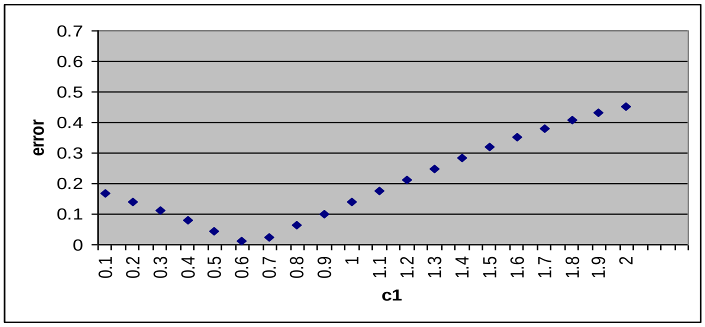
The value of Caccording to ideal values of DDC



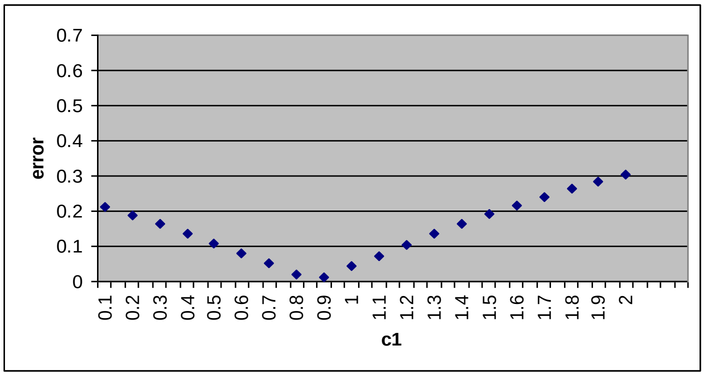
The value of Caccording to lower boundary values of DDC



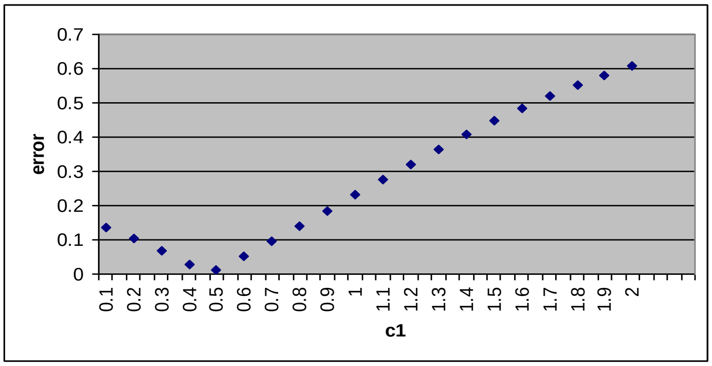
The value of Caccording to upper boundary values of DDC
Figure (11) The values of C in Hamad equation in case of d.d=1.2m



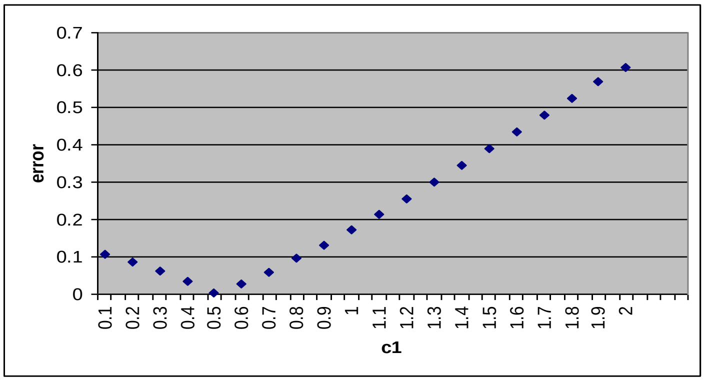
The value of Caccording to ideal values of DDC



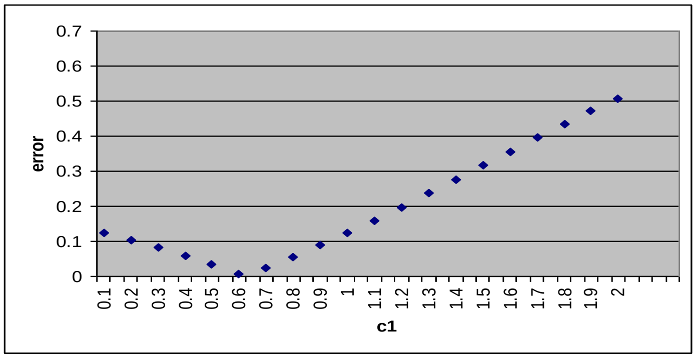
The value of Caccording to lower boundary values of DDC



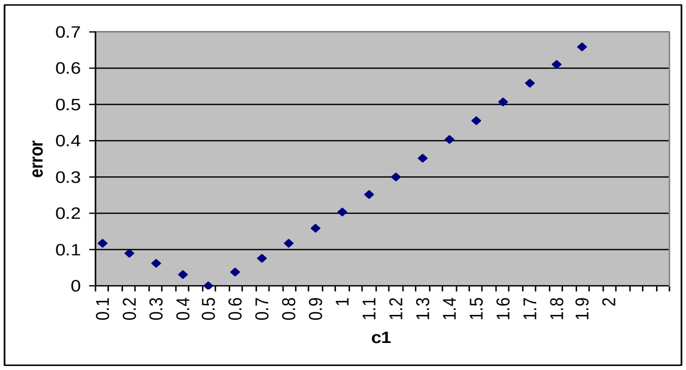
The value of Caccording to upper boundary values of DDC
Figure (12) The values of C in Hamad's equation in case of d.d=1.4m



The value of Caccording to ideally values of DDC



The value of Caccording tolower boundary values of DDC



The value of Caccording to upper boundary values of DDC

Figure (13) The values of C in Hamad's equation in case of d.d=1.7m

5.3. The difference in calculating space between laterals by using Hamad's equation

From **section 5.2,** C= 0.622. Then, the modified Hamad'sequation will be as follow:

$$L = \left(\frac{2\ \pi^{0.622} K t}{f\ \ln\left({}^{h_o}\!/\!{}_{h_t} \right)\ \ln\left({}^{L}\!/\!{}_{\pi r_o} \right)} \right)$$

While the original one is:

$$L = \left(\frac{2\ \pi\ K t}{f\ \ln\left({}^{h_o}\!/\!{}_{h_t} \right)\ \ln\left({}^{L}\!/\!{}_{\pi r_o} \right)} \right)$$

The space between laterals is calculated by usingboth theoriginal and the modifiedHamad's equation and recorded in **table (4).**The difference also calculated by the following equation and recorded in the same table.

$$difference(\%) = \left(\frac{L_2 - L_1}{L_2} \right) X10$$

Where, L_1= space between laterals calculated by the original equation**and**L_2= space between laterals calculated by the modified equation

Table (4) Difference incalculating space between lateralsby using Hamad's equation

	d.d =1.2			d.d =1.4			d.d =1.7		
	L.B	**IDD**	**U.B**	**L.B**	**IDD**	**U.B**	**L.B**	**IDD**	**U.B**
L_1	16.1	17.3	19.5	14.25	18.65	18.6	20.5	21	21.65
L_2	11.25	12.1	13.65	10	13.05	13	14.3	14.65	15.15
Difference(%)	-43.1	-42.9	-42.9	-42.5	-42.9	-43.1	-43.4	-43.3	-42.9

LB: Lower Boundary, **IDD:** Ideal Draw Down, **UB:** Upper Boundary

Table (4) illustrates that the average difference = - 43 % and the most values ofspace between laterals calculated by the original equation are not safe because they are more than the applied space in the area (L =15 m). On the other hand, they are more thanthe space between laterals calculated by the modified equation(that is the reason of the negative signof the error). As a result, it can be said thatthe original equation results in poor design and the modified equation results insafer solution.

6. SUMMARY AND CONCLUSIONS

In this research,three unsteady state equations,which are used in designing the subsurface drainage systems,were selected to be evaluated.These equations were Glover- Dumm's equation, Hamad'sequation and Luthien'sequation. The study resulted in some modifications in constants of both Glover-Dumm and Luthien'sequations to reach more economic results that matchtheconditions in Egypt. On the contrary, it stated some modifications in the constant of Hamad'sequation to reach safer results. Then, it can be said that before modification, both Glover-Dumm and Luthien'sequations result in over design but Hamad's equation**results in poor** design.

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