REVIEW OF FRICTION FORMULAE IN OPEN CHANNEL FLOW

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ABSTRACT

Due to the lack of analytic representation of frictional resistance in open channels, the traditional Manning or Chezy equation for steady uniform flow is usually assumed to be suitable as well as a practical representation of frictional resistance expected for unsteady flow.

No much attention has been given to any other frictional formula in application related to unsteady non-uniform flow or tidal phenomena in open channels and rivers.

The purpose of this research is to demonstrate the application of alternative equations of resistance, such as the rough turbulent formula, the Williamson equation and the Colebrook White equation. Differences between, and limitations of each formula are also presented.

An approach to the solution of Colebrook White formula in an explicit form in open channels is given. A comparative study between this formula and other explicit formulae is also presented.

1 REVIEW

1.1 The Chezy formula

In 1768, Antoine Chezy (Rouse, 1957), an engineer of the French Bureau of Bridges and Streets was given the task of designing a canal for the Paris water supply. He reasoned that the resistance would vary with the wetted perimeter and with the square of velocity, and the force to balance this resistance would vary with the area of cross section and with the slope. Therefore, he reasoned that $V^2 P/(A\ S)$ or $V^2/(R\ S)$ would be constant for any one channel and would be the same for any similar channel. His manuscript was not published until 1897, but his method gradually became known, and the square root of the preceding ratio came to be known as the Chezy coefficient

The formula can be written as:

$$V = C\sqrt{RS} \quad \text{.......} \quad (1)$$

where:

V : velocity of water;
C : Chezy's coefficient;
R : hydraulic radius; and
S : bed slope.

1.2 The Manning Equation

In 1891, another Frenchman, Flamant attributed wrongly to the Irishman R.Manning that C varies with the sixth root of R, although Gauckler in 1868 had proposed the same hypothesis for flat slopes and also Hagen in 1881 attributed the same concept to any slope (Rouse, 1957).

$$C = \frac{R^{\frac{1}{6}}}{n} \quad \ldots\ldots\ldots (2)$$

from which

$$V = \frac{1}{n} R^{\frac{2}{3}} \sqrt{S} \quad \ldots\ldots\ldots (3)$$

where: n is the characteristics of the surface roughness alone and the unit of length used is meter. In 1911 Buckly converted this equation to the foot second unit as

$$V = \frac{1.486}{n} R^{\frac{2}{3}} \sqrt{S} \quad \ldots\ldots\ldots (4)$$

This equation is known in the English speaking world as the Manning equation, although on the continent of Europe it is sometimes known as strickler's equation. The Manning equation has proved most reliable in practical and extremely popular in western countries In order to relate the Manning coefficient (n) and the equivalent particle size (k), Strickler's empirical formula (1923) was used as (Henderson, 1966):

$k = (n / 0.034)^6$ ………………………………………………………….(5)

in which (k) in feet

1.3 The Williamson formula

In 1951, James Williamson showed minor but reasonable adjustment to Nikuradse's results (Williamson, 1951) by correcting certain of Nikuradse's calculations and increasing the assumed grain sizes to allow for thickness of the varnish used to stick the grains, Fig. (1), to the pipe wall. He found that points representating Nikuradse's results fell mush closer than before to a straight line having slope 1:3. Further he plotted some more observations made by himself, Fig.(1), based on his experience with three large concrete line aqueducts and another of smaller which came under his design and supervision in the Galloway water power scheme in Scotland.

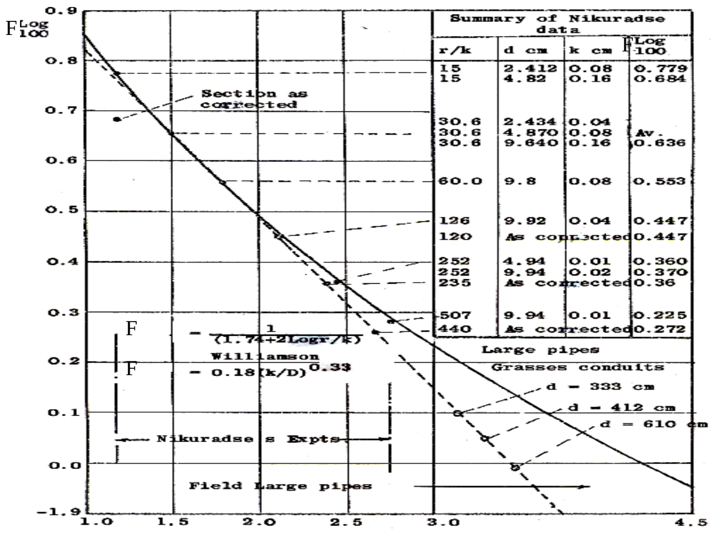

Figure 1. Correction of Nikuradse’s data (Williamson, 1951)

He found that they fell on a line having the same slope

The Williamson formula is given by:

$$F = 0.18\left(\frac{k}{d}\right)^{\frac{1}{3}} \qquad (6)$$

$$F = 0.113\left(\frac{k}{R}\right)^{\frac{1}{3}} \qquad (7)$$

where:

F : roughness coefficient;
k : equivalent particle size;
d : diameter of pipe; and
R : hydraulic radius

1.4 The Colebrook Equation

L. Prandtle and Von Karaman in Germany, and G.I. Tylor in expressing mathematical form the mechanism of turbulence linked the experimental investigation of Nikuradse (1932-1935) had proved a formula of the type (Colebrook, 1938):

$$\frac{1}{\sqrt{F}} = 2\log\left(\frac{0.112d}{y_1}\right) \qquad (8)$$

and showed the lower limit of the integration y_1, is a function of the wall particle size k, in the case of rough pipes in which the flow obeys the square resistance law, and is dependent on the density ρ, the viscosity μ and the shear stress at the wall τ in the case of smooth pipes.

In 1938, Cyril Frank Colebrook confirmed the substitution of values of y, in the foregoing equation and adopted the following resistance law

(a) Flow in hydraulically smooth pipes:

$$\frac{1}{\sqrt{F}} = -2\log\left(\frac{2.51}{R_e\sqrt{F}}\right) \qquad (9)$$

(b) Flow in hydraulically rough pipes:

$$\frac{1}{\sqrt{F}} = -2\log\left(\frac{k}{3.7d}\right) \qquad (10)$$

And he mentioned that the results of Nikuradse show complete agreement with above two laws provided certain limiting conditions are satisfied.

1.5 Transition Zones

The following formula is used to classify the different types of flow, either smooth, transition or rough turbulent $R_f = V^*k/\upsilon$
where:

V^* : shear velocity;
k : equivalent roughness height; and
υ : kinematics viscosity.

The roughness Reynolds number (R_f) may be expanded into:

$$\frac{V^*k}{\upsilon} = \sqrt{\frac{F}{8}}\,\frac{k}{R}\,\frac{VR}{\upsilon} \quad \ldots\ldots (11)$$

in which *F* is the resistance coefficient, *V* is the mean velocity and *R* is the hydraulic radius.

This expansion is a product of three dimensionless numbers, the resistance coefficient, the relative roughness, and Reynolds number.

The limit of R_f for transition zones has been given by Colebrook (1938) in pipes from 3 to 60, Keulegen and Patterson (1943) has provided this limit in open channels from 3.3 to 67. In order to remove any uncertainty, others have given the value of R_f for transition zone from 1 to 100. In this present study this value of R_f has been considered from 4 to 100 as given by Henderson (1970). A value of k equal to 0.03 ft (n=0.019) exhibited substantial transition zones in the river Forth of Scotland for 32 hour tidal wave input. The transition zone occurred at different places along the river from Stirling to Rosyth.

For k = 0.073 ft (n = 0.022) the transition zone has a negligible influence. A value of computed Reynolds number for transition zones in the river Forth of Scotland varied between 10^3 and 10^7 where the maximum Reynolds number was $1.5*10^8$ for rough turbulent flow conditions (Zidan, 1978).

An attempt to express mathematically the transition function for uniform sand roughness is rendered difficulty owing to the fact that the turbulent motion in the Wake behind the grain is complicated by the mutual interference, and the resistance mechanism made up of viscous and mechanical force which are difficult to separate. The exact distribution of the function will depend on the distribution of the roughness elements and it is mathematically indeterminate.

2 COLEBROOK- WHITE EQUATION

2.1 Implicit Colebrook-White Formula

A suggestion by C.M. White for transition formula which similar to those obtained experimentally for commercial pipes, was simply add together the lower limits of integration y, which satisfy the rough and smooth pipe laws, providing the general formula

For transition regime in which the friction factor varies with both *Re* and *k/d* , the equation universally adopted is due to Colebrook and White (1937) as:

$$\frac{1}{\sqrt{F}} = -2\ \log\left(\frac{k}{3.7d} + \frac{2.51}{R_e\sqrt{F}}\right) \quad (12)$$

where:

F : roughness coefficient;
k : equivalent particle size;
d : diameter of pipe; and
Re : Reynolds number

The Colebrook equation is transcendent and thus can not be solved in terms of elementary function. Some explicit approximate solutions have been proposed such as:

2.2Explicit Colebrook-White Formulae

The implicit nature of Colebrook White function (9) has sometimes acted against its adoption in preference to other friction formulae. In 1938 White gave an approximation to the logarithmic smooth turbulent element in the Colebrook White equation which was compatible in form with the original one. If Reynolds number is raised to an index could be accepted as a substitute for $Re\sqrt{F}$ in equation (9), then:

$$\frac{1}{\sqrt{F}} = 1.8\ \log\left(\frac{R_e}{6.8}\right) \quad (13)$$

or

$$\frac{1}{\sqrt{F}} = 2\ \log\left(\frac{R_e^{0.9}}{5.61}\right) \quad (14)$$

Barr Formula (1972) .This type of smooth law approximation has been used by Barr to form an explicit solution for the Colebrook-White equation

$$\frac{1}{\sqrt{F}} = -2\ \log\left(\frac{k}{3.7d} + \frac{5.1286}{R_e^{0.89}}\right) \quad (15)$$

and for more accuracy (Barr, 1975, 1976):

$$\frac{1}{\sqrt{F}} = -2\ \log\left(\frac{k}{3.7d} + \frac{\mathrm{Re}^{0.858+0.078\log \mathrm{Re}}}{2.735+0.575\log \mathrm{Re}}\right) \quad (16)$$

Some Explicit Formulae for Friction Factor in Pipe Flow

- Swamee- Jain (1976) : They proposed the equation covering the range of *Re* from $5*10^3$ to 10^7 and the relative roughness *k/d* between 0.00004 and 0.05 as:

$$F = \frac{0.25}{\left(\log\left(\frac{k}{3.7d} + \frac{5.74}{\mathrm{Re}^{0.9}}\right)\right)^2} \quad (17)$$

- Round (1980):

$$\frac{1}{\sqrt{F}}=1.8\ \log\left(\frac{\mathrm{Re}}{0.135\mathrm{Re}\left(\frac{k}{d}\right)+6.5}\right) \qquad \text{(18)}$$

- Barr (1981) (Beogradau and Brkie, 2012):

$$\frac{1}{\sqrt{F}}=-2\ \log\left(\frac{k}{3.7d}+\frac{5.1581\log\left(\frac{\mathrm{Re}}{7}\right)}{\mathrm{Re}\left(1+\frac{\mathrm{Re}^{0.52}}{29}\left(\frac{k}{d}\right)^{0.7}\right)}\right) \qquad \text{(19)}$$

- Haaland (1983) : Haaland formula is considered as one of the simpler formulae widely used in the application:

$$\frac{1}{\sqrt{F}}=-1.8\ \log\left(\left(\frac{k}{3.7d}\right)^{1.11}+\frac{6.9}{\mathrm{Re}}\right) \qquad \text{(20)}$$

- Serghides (1984): The equation was based on Steffensen's method. The solution involves calculation of three intermediate values *A*, *B*, and *C* first, and then substituting those values into the final solution:

$$F=\left(A-\frac{(B-A)^2}{C-2B+A}\right)^{-2} \qquad \text{(21)}$$

where:

$$A=-2\ \log\left(\frac{k}{3.7d}+\frac{12}{\mathrm{Re}^{0.983}}\right)$$

$$B=-2\ \log\left(\frac{k}{3.7d}+\frac{2.51A}{\mathrm{Re}}\right)$$

$$C=-2\ \log\left(\frac{k}{3.7d}+\frac{2.51B}{\mathrm{Re}}\right)$$

- Manadilli (1997) proposed the following expressions valid for *Re* ranging from 5235 to 10^8 for any value of *k/d*.

$$\frac{1}{\sqrt{F}}=-2\ \log\left(\frac{k}{3.7d}+\frac{95}{\mathrm{Re}^{0.983}}-\frac{96.82}{\mathrm{Re}}\right) \qquad \text{(22)}$$

- Gouder and Sommad (2006):

$$\frac{1}{\sqrt{F}} = 0.8686 \ln\left(\frac{0.4587\,\mathrm{Re}}{(S-0.31)^{\frac{S}{S+1}}}\right) \qquad (23)$$

where $S = 0.124\,\mathrm{Re}\frac{k}{d} + \ln(0.4587\,\mathrm{Re})$

- Avci and Karagoz (2009):

$$F = \frac{6.4}{\left(\ln(\mathrm{Re}) - \ln\left(1 + 0.01\,\mathrm{Re}\frac{k}{d}\left(1 + 10\sqrt{k/d}\right)\right)\right)^{2.4}} \qquad (24)$$

- Papaevangelou, Evangleids and Tzimopoulos (2010)

$$F = \frac{0.2479 - 0.0000947(7 - \log \mathrm{Re})^4}{\left(\log\left(\frac{k}{3.615\,d} + \frac{7.366}{\mathrm{Re}^{0.9142}}\right)\right)^2} \qquad (25)$$

3 COLEBROOK- WHITE EQUATION IN OPEN CHANNELS

3.1 Implicit Solution

For the purpose of analysis of the Colebrook White formula will be written as:

$$\frac{1}{\sqrt{F}} = c\,\log\left(\frac{k}{aR} + \frac{b}{\mathrm{Re}\sqrt{F}}\right) \qquad (26)$$

One of the first attempts to apply this formula to open channels flow was made in 1938 by A.P. Zegzhda in Russia (ASCE, 1963). He experimented with rectangular channels roughened with closely spaced sand grains of ten different sizes, the ratio of sand size to hydraulic radius varying from 0.125 to 0.2. His plot *F* versus *R* was similar to Nikuradse's plot for pipes. Where the Reynolds number was large enough so that *F* was constant and it was given by equation (9)

c =- 2.0, a = 11.55 and b = 0.0

Different values for a, b and are also given by different investigators based on experimental results, and finally the task force on friction factors in open channel, ASCE (1963) has recommended values of c =-2.0, a =12.0 and b = 0.0 for rough turbulent flow and b = 2.5 for partly rough turbulent flow providing the following two modified equations:

$$\frac{1}{\sqrt{F}} = -2\,\log\left(\frac{k}{12R}\right) \qquad (27)$$

$$\frac{1}{\sqrt{F}} = -2\,\log\left(\frac{k}{12R} + \frac{2.5}{\mathrm{Re}\sqrt{F}}\right) \qquad (28)$$

Equation (27) is the rough turbulent flow equation for open channel and corresponds to equation (10) for pipes and equation (29) is the modified Colebrook White equation for open channel which corresponds to equation (12) for flow in pipes.
The modified rough turbulent equation given by ASCE Task Committee (equation) is suitable for equivalent particle size $k >$ 0.1 ft. i.e. ($n > 0.024$) (Zidan, 1978)
No much attention has been given to the explicit form of Colebrook White equation for open channels as presented to pipe flow in literature

3.2 Explicit Solution

***Zidan's approach* (1978)**
Plotted data in Fig. (2) given by the university of Illionois demonstrate the turbulent zone corresponds closely to the Blasius Prandtl Von Karman curve. This indicates the law for turbulent in smooth pipes may be approximately representative of all smooth channels. This plot also shows that the shape of channel does not have an important influence on friction in turbulent flow

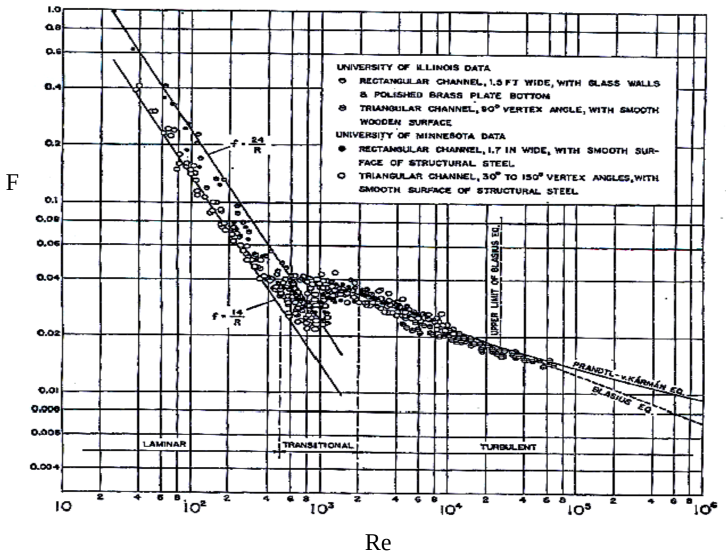

Figure 2. F-Re Relationship for flow in smooth channels
(Chow, 1971)

Bearing in mind that the smooth element in equation (28) can be substituted explicitly as a function of Reynolds number *Re* as has been done by White, and since the smooth turbulent flow in open channels can be represented by the smooth turbulent flow in pipes as mentioned before, so the smooth term in equation (28) can be replaced by an empirical term similar to that given by Barr

$$\frac{5.1286}{R_e^{0.89}}$$

or

$$\left(\frac{\mathrm{Re}^{0.858+0.078\log \mathrm{Re}}}{2.735+0.5715\log \mathrm{Re}}\right)$$ for more accuracy.

The two approaches for an explicit solution of Colebrook White equation can be written as:

$$\frac{1}{\sqrt{F}} = \frac{C}{\sqrt{8g}} = -2\,\log\left(\frac{k}{12R}+\frac{5.1286}{\mathrm{Re}^{0.89}}\right) \qquad (29)$$

or

$$\frac{1}{\sqrt{F}} = \frac{C}{\sqrt{8g}} = -2\ \log\left(\frac{k}{12R} + \frac{\mathrm{Re}^{0.858-0.078\log \mathrm{Re}}}{2.735+0.575\log \mathrm{Re}}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

Figs. (3) through (6) show simulation of 32 hours tidal wave at Westgrange in the river Forth using Manning , Chezy, Williamson and the modified Colebrook White (Zidan's approach) formulae respectively, for the case of rough turbulent flow (n=0.022). It is clear from these figures that the Manning, the Chezy, and the modified Colebrook White formulae are suitable for rough turbulent flow and there is no appreciable error between these formulae, Table (1), and have given results near to the actual.
Use of the Williamson formula (7) in the computation produced lower frictional resistance than the other rough turbulent equations. This being noticeable in producing higher tidal range and appreciably larger size of hump, Fig. (5), this due to smaller relative roughness. *2R/k* varies between 200 and 3500 in the Forth. Williamson stated that Nikurades rough pipe law would not apply *d/k* value above 500 or *2R/k* above 250.

The percentage difference ratio of the Manning, Chezy and Williamson formulae to the modified Colebrook White equation *(the Zidan approach*) for water levels ,tidal range, and water discharge at two stations: (1) upstream river Forth (Westgrange) and the other (2) at the downstream (Rosyth) is seen in Table (1) for n =0.022 where the transition zones are negligible. As the value of n is reduced to be less than 0.022 or k < 0.073 ft . points of the tidal cycle fall within the smooth-rough turbulent transition zones. It would seem that the application of either the Manning, the Chezy, rough turbulent, and the Williamson equations will produce error in the computations. These errors become substantial for the value of Manning's n less than 0.02 or k < 0.04 ft.

Table 1. Percentage difference ratios between Manning, Chezy, Williamson formulae to the Zidan's approach for water levels, tidal range and discharge. k =0.073 ft. n =(0.022)

Equation	Stn.	Min W.L	Max W.L	Tidal Range	Ave. Min W.L	Ave. Max W.L	Ave. Tidal Range	Ebb Discharge	Flood Discharge
Manning	**1**	**4.35**	**0.79**	**1.88**	**0.39**	**0.25**	**1.46**	**0.70**	**1.81**
	2	**5.69**	**0.12**	**0.22**				**0.34**	**0.29**
Chezy	**1**	**4.68**	**0.52**	**1.69**	**0.0**	**0.25**	**1.55**	**0.98**	**1.53**
	2	**6.1**	**0.0**	**1.41**				**0.04**	**0.45**
Williams.	**1**	**3.68**	**2.88**	**3.29**	**9.55**	**3.25**	**3.48**	**6.36**	**11.63**
	2	**4.47**	**3.41**	**3.66**				**0.22**	**6.12**

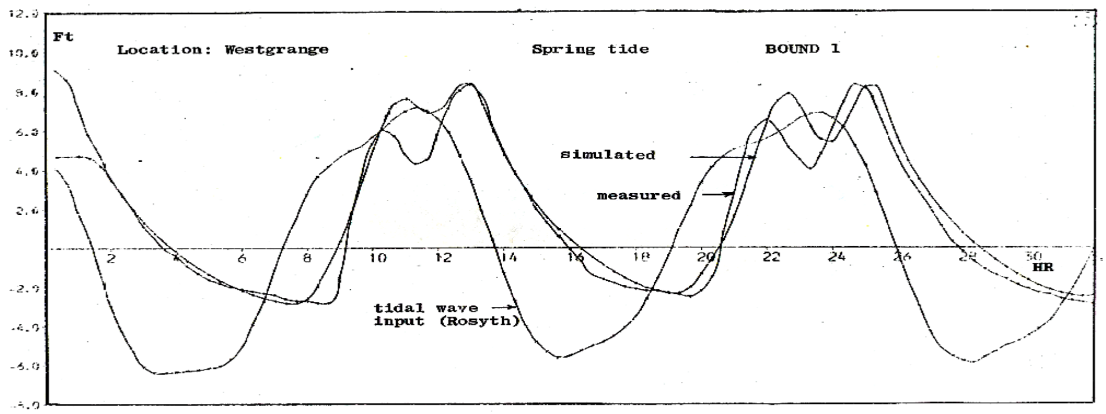

Figure 3. Simulation using Manning's formula.

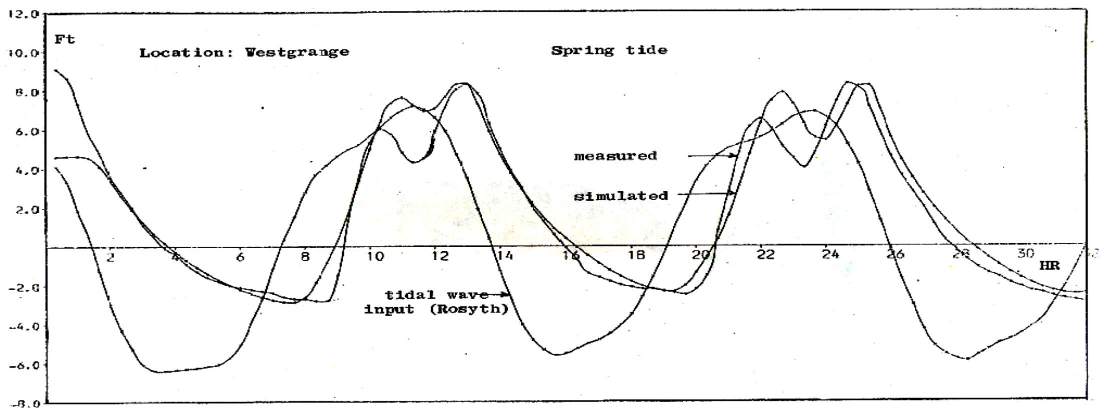

Figure 4. Simulation using Chezy's formula.

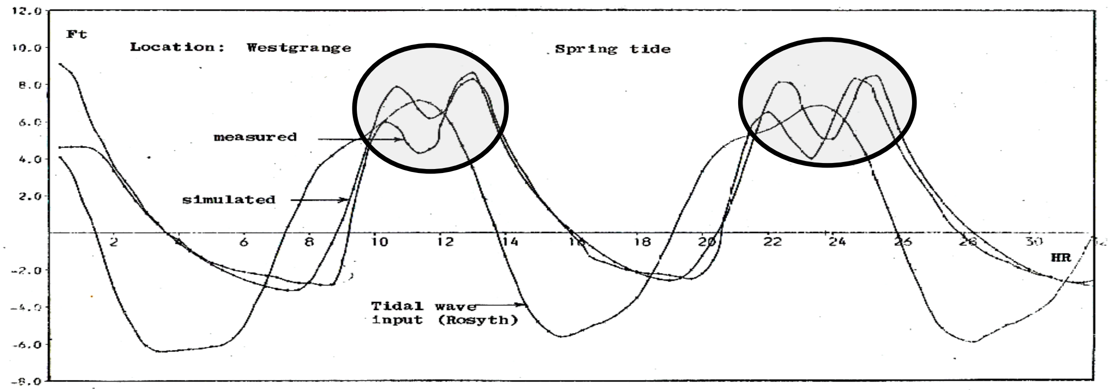

Figure 5. Simulation using Williamson's formula showing larger "humps".

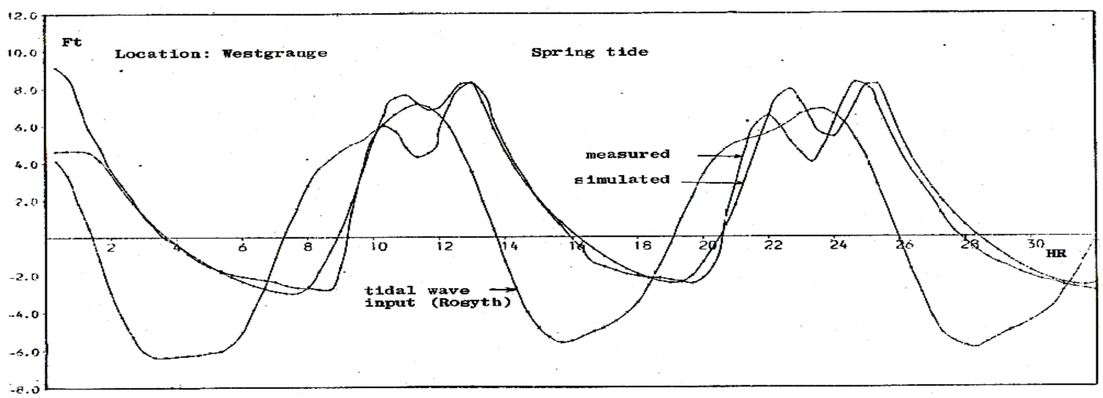

Figure 6. Simulation using modified Colebrook White's formula (Zidan's approach)(Zidan, 1978)

Yen's approach (1991) could be written as:

$$\frac{1}{\sqrt{F}} = \frac{C}{\sqrt{8g}} = -2\ \log\left(\frac{k}{12R} + \frac{1.95}{\mathrm{Re}^{0.9}}\right) \qquad (31)$$

Fenton's approach (2010) could be written as:

$$\frac{1}{\sqrt{F}} = \frac{C}{\sqrt{8g}} = -2\ \log\left(\frac{k}{12R} + \left(\frac{2}{\mathrm{Re}}\right)^{0.9}\right) \qquad (32)$$

Both Yen and Fenton formulae are applicable for *Re* > $3*10^4$ and the relative roughness *k/R* < 0.05.(Fenton, 2010)

Reynolds number term in the numerator $2^{0.9} = 1.87$ in Fenton equation, for wide channel not 1.95 as given by Yen. He stated that this value would be smaller for narrow channels.

4 COPARITIVE STUDY

For the sake of comparison between Yen's approach and Zidan's approach, it is assumed that the hydraulic radius (*R*) varies between 1 m and 20 m and values of Reynolds number R_e varies between $3*10^4$ and 10^8. The modified Colebrook White equation given by Zidan exhibits smaller values than the corresponding ones using either Yen or Fenton approach. Table (2) gives a percentage error between both the Yen approach with respect to Zidan's approach which has been tested using actual data of the river Forth of Scotland The results of this approach has been compared with the corresponding ones of the implicit equation given by ASCE Task Force on Friction Factor and showed an error of about 0.30 %. Fenton gives a slight higher values than the corresponding ones of Yen formula, for all values of R_e as given by Yen and Fenton (from $3*10^4$ to 10^7). This difference may not exceed 1 % in case of transitional turbulent flow. Increasing the value of hydraulic radius *R* and Reynolds number R_e will decrease the percentage between the two equations. Increase the value of *k* will decrease this percentage, Table (4)

Table 2. Percentage difference ratios of the Yen approach to the Zidan approach for turbulent flow.

Reynolds No. Case	$3*10^4$	10^5	10^6	10^7	10^8
k = 0.03 ft.					
Maximum error (%)	**12.73**	**9.42**	**2.66**	**0.51**	**0.17**
Average error (%)	**9.22**	**5.53**	**1.14**	**0.18**	**0.092**
k. = 0.073ft.					
Maximum error (%)	**9.98**	**6.97**	**1.6**	**0.23**	**0.12**
Average error (%)	**4.67**	**2.63**	**0.53**	**0.06**	**0.04**

Table (3) gives the differences between the three approaches for transition turbulent flow, Reynolds number ranges between 500 and 2000. It is noticed generally that the percentage error increases than the case of rough turbulent and decreases with the increasing value of R_e .Also it noticed that there a difference between Yen Formula and Zidan approach reaches to about 20.1 % at k = 0.03 ft. and 19.8 % at k = 0.073 ft. for R_e = 2000 and hydraulic radius R = 20

Table 3. Percentage difference ratios of the Yen approach to the Zidan approach for transition turbulent flow (Re = 500 to 2000).

Reynolds No. / Case	500	1000	1500	2000
k = 0.03 ft.				
Maximum error (%)	25.9	20.30	20.21	20.11
Average error (%)	25.33	19.74	19.25	18.88
k = 0.073 ft.				
Maximum error (%)	25.72	22.49	21.16	19.81
Average error (%)	24.68	21.07	10.24	17.40

5 CONCLUSIONS

The Manning and Chezy equations are suitable for simulation of resistance in flow which are fully rough turbulent. This clear from results obtained in table (1) where the condition of flow is nearly rough turbulent, and the two formulae have given results near to the actual. Use of the Williamson formula, equation (7) in the computation produced lower frictional resistance than other rough turbulent equations. This being noticeable in producing higher tidal ranges and appreciably large size of hump in tidal wave due to the smaller relative roughness. In the river Forth *2R / k* varies between 200 and 3500. Williamson stated that the Nikuradse's rough pipes law would not apply *2R / k* value above 250.

The Colebrook White equation covers not only the transition region but also the fully developed smooth and rough open channel as k = 0.0 this reduces the equation for smooth channel as $k \to \infty$ it forms an equation for rough channels. Unlike the Manning or Chezy equation which should be applied for rough zone only.

The difference between the implicit solution of equation (28) and the explicit solution of equation (29) is about 0.3%. The advantage of Colebrook White equation is any big error in *Re, k, or R* gives a small error in the value of C due to its logarithmic nature.

The Manning formula like the Chezy or the Wlliamson formula is suitable for unsteady varied flow or tidal computation under rough turbulent condition n > 0.023 or k > 0.1 ft.. For transition turbulent the two equations are no longer suitable, unless these equations are dependent on Reynolds number R_e

The Manning and Chezy equations produce slight difference in the computation at some places , bearing in mind the two roughness coefficients in the two equations are given by $C =(1/ n) R^{1/6}$. The difference may indicate that the relationship between n and C may be a function of cross sectional shape or the above relationship is not always proportional to $R^{1/6}$

As the value of Manning's n is reduced to less than 0.022 or k < 0.073 ft. points of tidal cycle fall within the smooth–rough transition zones .It would seem that the application of either the Manning , Chezy or Williamson equation will produce errors in the computation. These Errors become substantial for the value of Manning ' s n less than 0.02 or $k <$ 0.04 ft.

The Colebrook White explicit equation

$$\frac{1}{\sqrt{F}} = \frac{C}{\sqrt{8g}} = -2\ \log\left(\frac{k}{12R} + \frac{5.1286}{R_e^{0.89}}\right)$$

seems to be suitable for unsteady varied flow and tidal computations in open channels. It has two advantages, the first being its logarithmic form, i.e for any large error in estimating the value of k will produce only small error in the evaluating of *F* or *C*. Secondly it is suitable for coping with the transition zones especially if these zones are of substantial duration, i.e. more physically correct. It could also be concluded that the partly rough pipes flow will not behave exactly as partly rough open channel flow.

It will be useful for future work to study the behavior of the friction factor *F* or *C* towards Reynolds's number *Re* in natural channels like these studies which had been made by Nikuraadse based on experimental pipes. Since the relation between Reynolds's number *Re* and friction factor *F* or *C* could be accepted as exponential relationship as mentioned before. It will be useful to trace the transition zones, using formula (11) in natural rivers or open channels and to draw the relationship between *F* or *C* and *Re* for an average relative roughness to find the most accurate value for the Index, which could be used for the substitution in the explicit modified Colebrook White equation.

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