ESTIMATION OF DAM BREACH WIDTHS USING A NEURO-FUZZY COMPUTING TECHNIQUE

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ABSTRACT

Accurate prediction of dam breach width is crucial in dam risk assessments because it significantly influences peak breach outflow, inundation levels, and flood arrival time. This paper investigates the abilities of Adaptive Neuro-Fuzzy Inference system (ANFIS) method to improve the accuracy of embankment breach width estimation. Different ANFIS models comprising various combinations of variables including reservoir storage, height above breach invert, dam width and dam materials are developed to evaluate degree of effect of each of these variables on breach width. Historical data from 79 embankment dam failures are used in the development and testing of the ANFIS model. A comparison is made between the estimates provided by the ANFIS model and the available regression equations (RE). Uncertainty analysis and several statistical measures are also used to evaluate the performance of the ANFIS models. The results indicated the potential of the ANFIS model to be used as a predictive tool for estimating the average breach width of embankment dams.

Keywords: Breach width; Embankment dam; Uncertainty analysis, neuro-fuzzy.

1. INTRODUCTION

Embankment dams are built for many purposes (water supply, power generation, irrigation, recreational and fishery improvement, etc.) and preferred under certain circumstances, especially when sufficient materials are available near the dam site, the foundation is pervious, and the ratio of dam length to height is high. The main problem facing this kind of dam is piping or overtopping that may cause erosion of materials and ultimately breaching of the embankment. Accurate estimations of breach characteristics are needed as a basis in dam risk assessments. In order to carry out an embankment failure analysis the average breach width in the dam is one of the key parameters that should be accurately estimated because it influences the severity of failure and affects the magnitude of the peak discharge. The breach shape of an embankment dam is assumed to vary from triangular to trapezoidal as the breach progresses, Wahl [24]. The average breach width (B_{av}) is one-half the sum of the trapezoid top and bottom widths. Methods of estimating B_{av} are based on either case study data from past dam failures or physically-based models. In practice, the most widely applied methods to predict B_{av} are the regression equations (RE) based on regression analysis of recoded data from embankment dam failures, e.g. the Bureau of Reclamation [1], Von Thun and Gillette [23], and Froehlich [8 and 10]. RE provide simple and convenient algorithms, under the assumption of strong linear relationships between the input and output variables and when detailed simulations are not required. If these assumptions are violated then the linear regression approach leads to biased relationships. Uncertainty is also included in determining the reservoir water volume and the breach height at the time of failure that will be used to predict the breach width. It is not possible to consider such variations in the coefficients through regression analysis. The Adaptive Neuro-Fuzzy Inference system (ANFIS), on the other hand, can provide an alternative methodology for considering such uncertainties through vaguely defined membership functions. More details about the ANFIS modeling will be given in subsequent sections. ANFIS modeling is effectively utilized in applications

ranging over perhaps all branches of engineering. However, there is currently no solution to predict B_{av} using this technique.
The main objective of the present paper is the development of ANFIS models to predict B_{av} as an alternative to the RE. The available case studies of embankment dam failures presented by Froehlich [10] constitute the basis for the development of these ANFIS models. This general objective includes: (1) developing ANFIS models for predicting B_{av} using first-order Sugeno fuzzy inference engine; (2) evaluating the ANFIS models by comparing its estimates with the observed values and estimates of best available RE for predicting B_{av}; (3) evaluating the ANFIS models using some basic statistical parameters and (4) performing inter-comparisons between the estimates of ANFIS models in order to obtain the best model for predicted B_{av}.

2. REVIEW OF AVAILABLE APPROACHES

Several physically-based models are available in literature to simulate the breach of embankment dams, e.g. Cristofano [2]; Ponce and Tsivoglou [17]; Fread [5, 6]; Visser [22]; Froehlich [9]; Hanson et al. [14]. They rely on sediment erosion and water flow formulas and generally suffer from insufficient understanding of breach development, Wahl [24]. As the material changes, more uncertainties become included in the overall breaching process. For more practical and easily applied models, many researchers gathered detailed case studies of breached embankment dams and developed expressions to predict the characteristics and consequences of the breach. From those studies, Johnson and Illes [13], Singh and Snorrason [19] and The Federal Energy Regulatory Commission [FERC 4] recommended a range for the breach width as a linear function of the dam height (h_d). Froehlich [7] used nondimensional analysis and developed an equation that estimates the average breach width as a function of the non-dimensional reservoir storage (S^*). Von Thun and Gillette [23] used the data of MacDonald and Langridge-Monopolis [15] and Froehlich [7] and proposed a relation for estimating B_{av} knowing the depth of water at the dam at time of failure (h_w) and a coefficient (C_b) that depends on the reservoir storage. Later on, Froehlich [8] published a revised equation that has better estimated coefficients to predict B_{av}. The independent variables in this equation are the volume of water stored above the breach invert at time of failure (V_w), the breach height (h_b) and a factor (K_0) that accounts for failure mode. Wahl [24 and 25] provided a summary of the available RE for predicting the breach width, performed an uncertainty analysis and compared state-of-the-art prediction equations. Wahl stated that Froehlich's [8] equation had the best prediction performance for cases with observed breach widths less than 50 m. In 2008, Froehlich [10] proposed another equation that will likely be accurate enough in application to estimate B_{av} as a function of $V_w^{1/3}$ and K_0. Most of the RE are derived under the assumption of linear relationships between the input and output variables. The results of the available RE vary widely depending on the assumptions, variables, subsets of data used in their formulation, and internal uncertainties that are not taken explicitly into consideration. The linear regression approach assumes that the scatter of points around the best-fit line is approximately Gaussian and has the same standard deviation all along the line, and the data points are independent of one another. Based on the fuzzy set theory introduced by Zadeh [26], Elmazoghi [3] developed a fuzzy logic model to estimate the B_{av} and the simulation results indicated the potential of the fuzzy logic model to be used as a predictive tool. From the above cited researches it can be inferred that models based on conventional mathematical tools (e.g., regression) require several assumptions to deal with non-linear and uncertain systems. Hence, application of ANFIS modelling offers an alternative that allows the modeller to include imprecise data and parameters without the need for any assumption.

3. AN ANFIS
3.1 MATERIALS AND METHODOLOGY

A neuro-fuzzy system combines fuzzy logic with neural networks in order to have better results for systems possessing nonlinear behaviour, uncertain parameters and data. The adaptive neuro-fuzzy inference system ANFIS can be described as a fuzzy inference system (FIS) equipped with a training algorithm, Jang [11]. A FIS consists of: (1) IF-THEN fuzzy rule base, (2) membership functions to be used in the fuzzy rules and (3) a reasoning mechanism, which performs the inference procedure upon the rules to obtain the desired output. The ANFIS uses a hybrid-learning rule combining back-propagation, gradient-descent, and a least-squares algorithm to identify and optimize the Sugeno system's parameters. In this study, the breach width (B_{av}) can be characterized as a function of the water height above breach invert h_w, the reservoir storage S, average width of the dam W, and presence or absence of core $C\ or\ NoC$.

Two alternatives were used to train the ANFIS models as follows:

1. Train one model using all available data from dam failure case studies with and without classification. The available information with complete data concerning the height of water above the breach invert (h_w), the reservoir storage (S) and the average breach width (B_{av}) comprises 79 case studies. This data is subdivided into two groups where one is used in the training phase of the model and the other is used in the testing phase of the model.
2. Train more than one model after classification of the data according to the available information about: the height of water above breach invert (h_w), dam height (h_d), reservoir storage (S), dam construction material (zoned, cored or homogeneous, without core C, No C), and average width of the dam (W). The classification of dam types was based on:
   - Availability of all hydraulic information about (h_w, h_d, S, W, C or No C).
   - Size of the dam. According to International Committee Of Large Dams, ICOLD, If h_d < 15 m then the dam is a small dam. If h_d > 15 m or between 10 and 15 m but $S \geq 1$ Mm^3 then the dam is a large dam, Singh [20].

With the purpose of performing the training and testing phases of the different models, the number of observations with complete data according to the above classifications are counted and taken into consideration. Different scenarios with different inputs and number of observations are proposed in this study as given in Table 1. The number of case studies in each scenario depends on the availability of equal information about the selected hydraulic variables in that scenario. The relationships between the breach width and the input variables for the various scenarios can be expressed by:

$B_{av} = f(h_w, S)$
$B_{av} = f(h_w, S, W)$
$B_{av} = f(h_w, S, W, C\ or\ NoC)$

Table 2 presents the ranges and the linguistic labels of the fuzzy membership functions (MF) for the input variables. From 3 to 7 curves are generally appropriate to cover the required range of a fuzzy variable, Ross [18]. Triangular and Gaussian membership functions are used in the ANFIS models. It takes several trials in order to reach the optimum number and shape of membership functions that result in reliable estimates for the output. Figures 1, 2 and 3 show the MF for the different ANFIS models developed in this study to predict the average breach width.

Table 1 Scenarios used to Train the ANFIS models to predict the average breach width (B_{av}) of embankment Dams

Scenarios	Inputs	Custom ANFIS
S1	h_w, S 79 case studies	Membership function type: triangular MF Number of memberships: (6, 6) functions Learning algorithm: Hybrid learning algorithms Sugeno type-system: First order Output type: Linear
S2	h_w, S, W 56 case studies	Membership function type: triangular MF Number of memberships: (4, 4, 4) functions Learning algorithm: Hybrid learning algorithms Sugeno type-system: First order Output type: Linear
S3	h_w, S, W, C or NoC 56 case studies	Membership function type: Gaussian MF Number of memberships: (4, 4, 2, 4) functions Learning algorithm: Hybrid learning algorithms Sugeno type-system: First order Output type: Linear

Table 2 The linguistic labels for fuzzy membership functions of input variables

The variable	The linguistic variable	
Depth above breach ***(h_w)*** m Range of data 1.68 to 77.4 m	VSH SH M MH H VH	Very Short Short Medium Medium High High Very High
Reservoir storage ***(S)*** $\times 10^6$ m^3 Range of data 0.0234 to 356	VL L M MH H VH	Very Low Low Medium Medium High High Very High
Existence or absence of core Range of data 0 or 1	C No C	Dam With Core Dam Without Core
Dam average width ***(W)*** m Rage of data 7.6 to 250 m	VS S M MH H VH	Very Short Short Medium Medium High High Very High

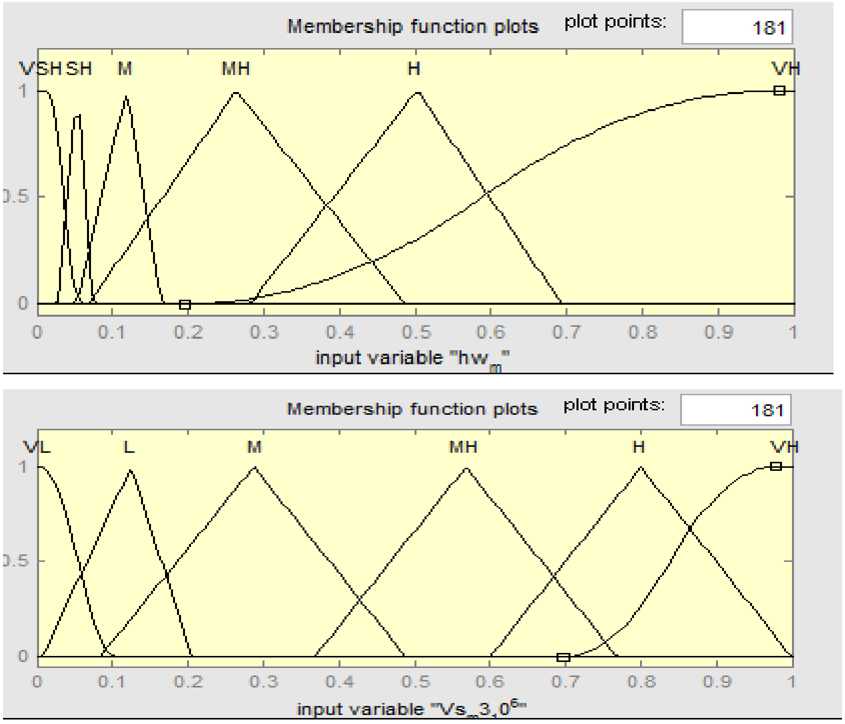


Fig. 1 Membership functions of input variables, scenario: $B_{av} = f(h_w, S)$

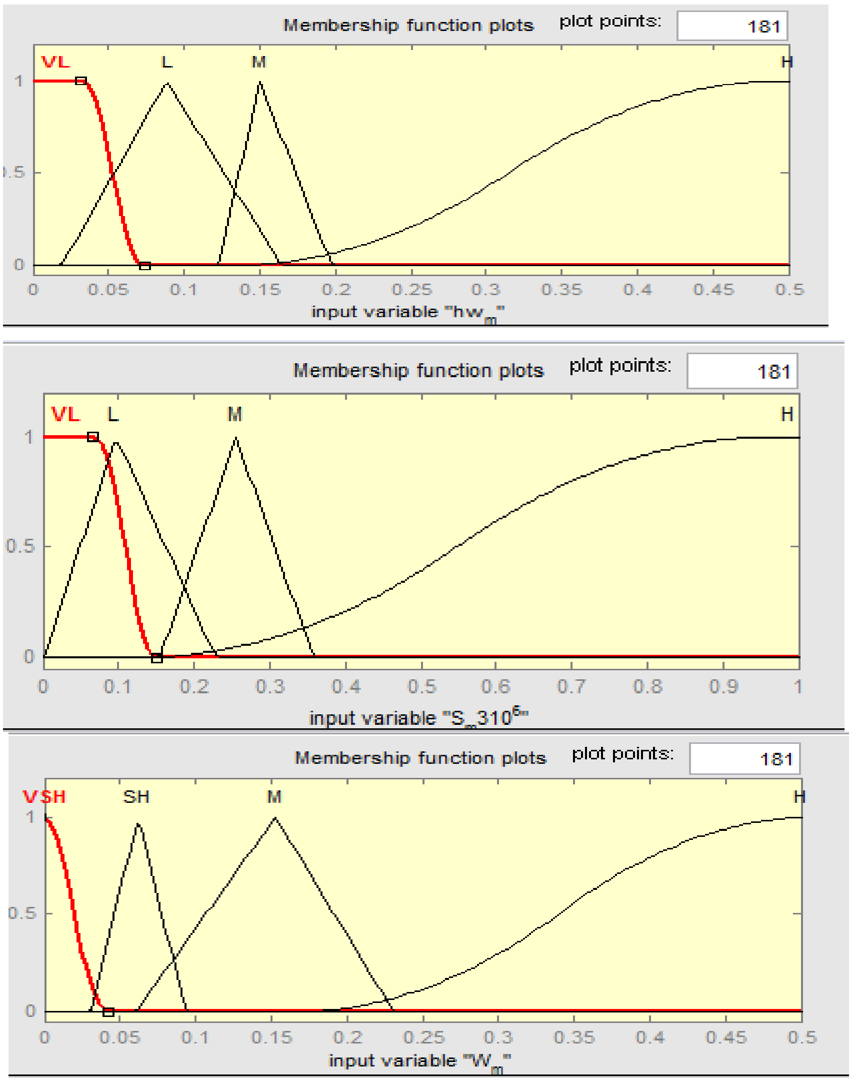


Fig. 2 Membership functions of input variables, scenario: $B_{av} = f(h_w, S, W)$

To illustrate the ANFIS method, consider the first model including two inputs: the height of water above breach invert (h_w) and the reservoir storage (S), and one output, the average breach width (B_{av}). Suppose that the rule base contains two fuzzy IF-THEN rules. For the first-order Sugeno FIS, Takagi and Sugeno [20], the two rules can be expressed as:

Rule 1: $if\ h_w\ is\ A_1\ and\ S\ is\ B_1\ then\ f_1 = p_1 h_w + q_1 S + r_1$
Rule 2: $if\ h_w\ is\ A_2\ and\ S\ is\ B_2\ then\ f_2 = p_2 h_w + q_2 S + r_2$

where A_1, A_2 and B_1, B_2 are the membership functions (MF) for inputs h_w and S, respectively; p_i, q_i and r_i (i= 1 or 2) are linear parameters in the consequent part of the first-order Sugeno FIS. These parameters have to be determined in the training process besides premise parameters which belong to membership functions. The architecture of the ANFIS is illustrated in Fig. 4. The ANFIS architecture consists of five layers, namely, a fuzzy layer, a product layer, a normalized layer, a defuzzy layer and a total output layer.

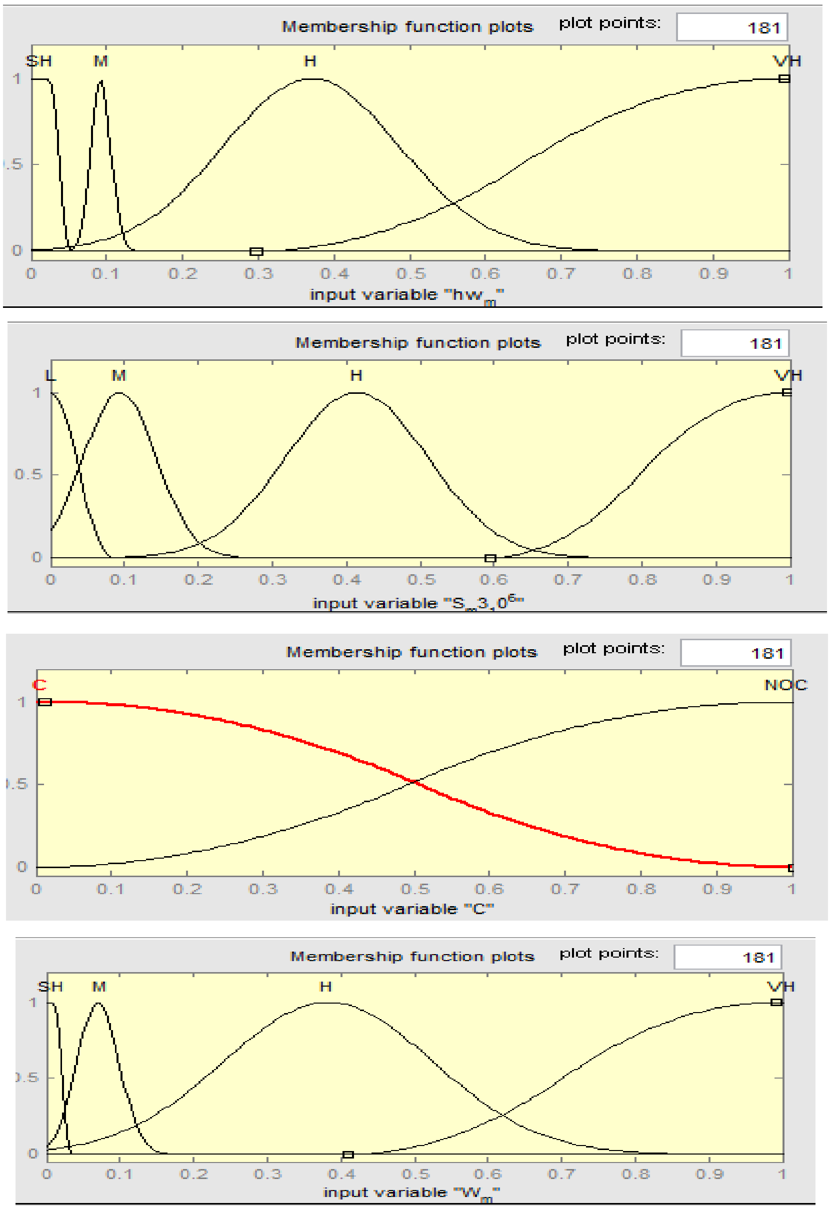


Fig. 3 Membership functions of input variables, scenario: $B_{av} = f(h_w, S, W, C\ or\ NoC)$

Layer 1: consists of adaptive nodes that assign membership grades for linguistic labels (such as small, medium, large, etc.) depending on premise parameters. For generalized bell membership functions, the node function is given by:

$$O_{1,i} = \mu_{A_i}(h_w) = e^{\frac{-(x-\delta_i)^2}{2\sigma_i^2}}, \qquad \text{for } i=1, 2 \text{ or}$$

$$O_{1,i} = \mu_{B_{i-2}}(S) = e^{\frac{-(y-\delta_i)^2}{2\sigma_i^2}}, \qquad \text{for } i=3, 4$$

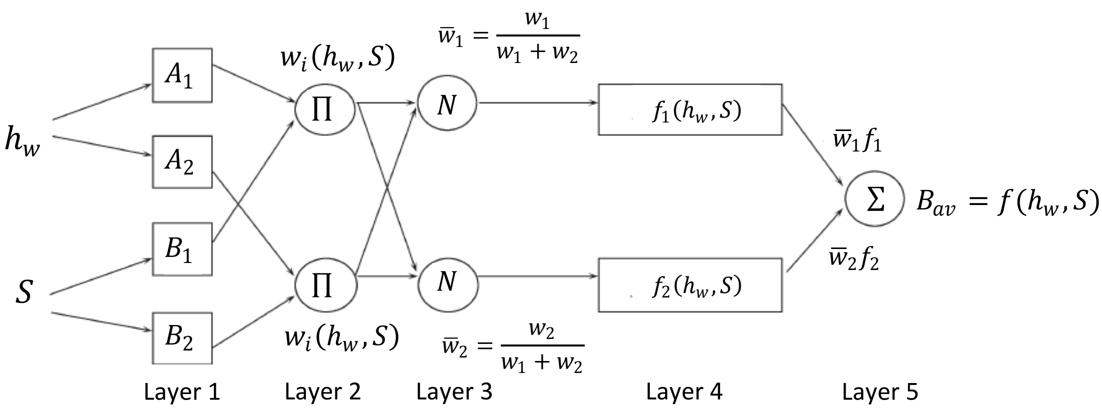


Fig. 4 Architecture of ANFIS

Where: $O_{1,i}$ is the membership grade of a fuzzy set A (=A_1, A_2, B_1, B_2) and it specifies the degree to which a given input h_w (or S) satisfies the quantifier A, $\{\delta_I, \sigma_I\}$ is the parameter set of the membership functions in the premise part of fuzzy IF-THEN rules that adjusts the shapes of the membership functions. Figure 5 shows a part of the rule base of one of the ANFIS models.

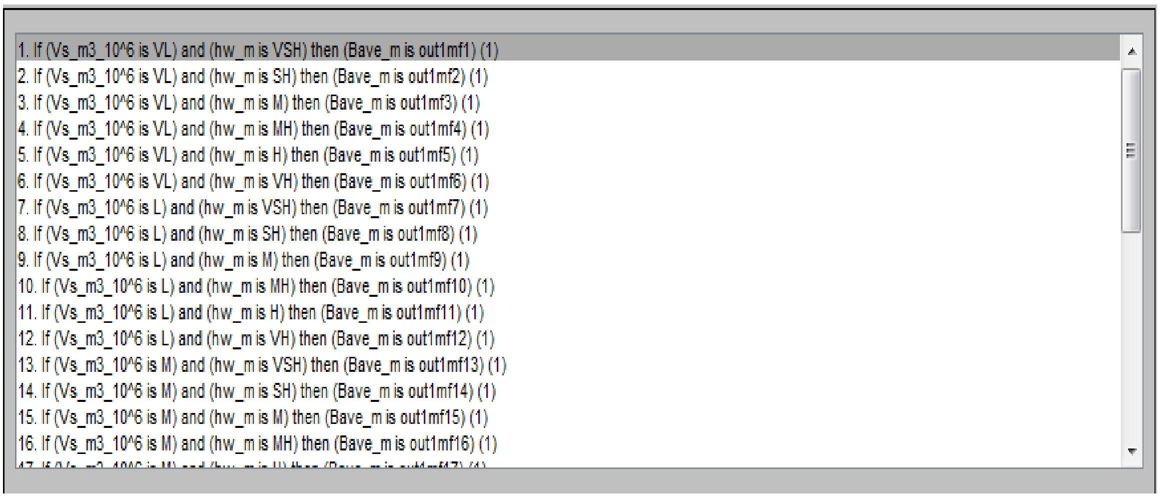


Fig. 5 Part of the rule base of one of the ANFIS models

Layer2: nodes in layer 2 are fixed nodes designated Π, which represent the firing strength of each rule. The output of each node is the fuzzy AND (Minimum) of all membership degrees.

$$O_{2,i} = w_i = \mu_{A_i}(h_w) \times \mu_{B_i}(S), i = 1, 2$$

Layer3: outputs of layer 3 are the normalized firing strengths. Each node is a fixed rule labeled N. The output of the i^th^ node is the ratio of the i^th^ rule's firing strength to the sum of all the rules firing strengths:

$$O_{3,i} = \overline{w}_i = \frac{w_i}{\sum_i w_i}, i = 1, 2$$

Layer4: adaptive nodes in layer 4 calculate the rule outputs based upon consequent parameters using the function:

$$O_{4,i} = \overline{w}_i f_i = \overline{w}_i (p_i h_w + q_i S + r_i), i = 1, 2$$

where $\overline{w}_i$ is the normalized firing strength obtained from layer 3, and (p_i, q_i and r_i) is the consequent parameter set of the node.

Layer5: the single node in layer 5, labeled $\sum$, calculates the overall ANFIS output by summing all the incoming signals. The defuzzification process transforms each rule's fuzzy results into a crisp output:

$$O_{5,i} = B_{av} = \sum_i \overline{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i}$$

Training the ANFIS is a two-pass process over a number of epochs. During each epoch, the node outputs are calculated up to layer 4. At layer 5, the consequent parameters are calculated using a least-squares regression method. The output of the ANFIS is calculated and the errors propagated back through the layers in order to determine the premise parameter (layer 1) updates.

3.2 DEVELOPMENT OF ANFIS-BASED MODEL

The criterion chosen for the development of the ANFIS model as shown in Table 1 was based on the selection of the following: Membership Function Type, Number of Membership Functions, Learning Algorithm, Epoch Size and Data Size (training, testing). Data normalization is performed because it can speed up training time by starting the training process for each feature within the same scale. It is especially useful for modeling application where the inputs are generally on widely different scales. Min-Max Normalization method rescales the inputs or outputs from one range of values v to a new range of values new_v. More often, the features are rescaled to lie within a range of 0.0 to 1.0. The rescaling is often accomplished by using a linear interpretation formula such as:

$$new\_v = \frac{v - min_x}{max_x - min_x}$$

Min-max normalization has the advantage of preserving exactly all relationships in the data. It will encounter an "out-of-bounds" error if a future input case for normalization fails. Once the output values (the average breach width, B_{av}) were obtained from the model, transform them back to the original scale using the inverse transformation: $B_{av} = min_x + v_{ANFIS}(max_x - min_x)$. The modeling criterion adopted is to effectively tune the membership functions so as to minimize the output error measure and maximize performance index. The Adaptive Neuro-Fuzzy Inference Systems models were trained and tested with the ANFIS editor. The ANFIS toolbox employed is the MATLAB ®V7.10 (R2010a). The models were developed using the following steps at the ANFIS Graphical User Interface (GUI): (1) Obtaining training data, (2) Data sizing, (3) Data partitioning, (4) Loading the data sets.

3.3 EXAMINATION OF THE RELIABILITY OF THE ANFIS MODEL

The performances of the developed ANFIS models were evaluated by using a variety of standard statistical performance evaluation measures. To limit the space in this paper, only four statistical performance indices are presented: Mean Absolute Error (MAE), Root mean square error ($RMSE$), Nash-Sutcliffe efficiency (E) and the coefficient of determination (R^2). These statistical parameters are calculated during the training and the testing phases using the observed and estimated breach width data from the ANFIS and RE models.
1. Mean Absolute Error **(**MAE): measures the mean absolute error between The observed and the predicted values.

$$MAE = \frac{\sum_{i=1}^{n}|(x_i - x_i^*)|}{n}$$

Where: x_i= The observed value, x_i^*= The predicted value and n= the total number of observations.

2. Root mean square error **(**$RMSE$): measures the root square of the mean error

$$RMSE = \sqrt{\frac{\sum_{I=1}^{N}(x_i - x_i^*)^2}{n}}$$

3. Nash-Sutcliffe efficiency (coefficient of efficiency**,** E**)**
The efficiency factor (E) proposed by Nash and Sutcliffe [16] is defined as follows:

$$E = 1 - \left(\frac{\sum_{i=1}^{n}(x_i - x_i^*)^2}{\sum_{i=1}^{n}(x_i - \bar{x})^2}\right)$$

Where: $\bar{x}$ is the average of observed values.
The range of E lies between 1.0 (perfect fit) and $-\infty$. However, an efficiency of lower than zero indicates that the mean value of the observed phenomenon would have been a better predictor than the model. The uncertainty analysis that one can express a confidence band around the predicted value of a parameter as: $\{\hat{x} \times 10^{-\bar{e}-2S_e},\ \hat{x} \times 10^{-\bar{e}+2S_e}\}$. The use of $\pm 2S_e$ approximately yields a 95% confidence band. For the calculation of the above statistics, the Microsoft Excel Statistical Package is employed. The Root Mean Square Error is the measurement of the models performance during the training phase. The *RMSE* indicates how "close" one data series is to another. In our case, the data series are the Target (actual) output values and the corresponding predicted output values generated by the model.

4. ANALYSIS OF THE RESULTS

Figure 6 compares the results of the best ANFIS model with observation data for the first scenario using h_w and S as inputs to predict the average breach width B_{av}. The figure shows the estimates of two ANFIS models: (1) a model without classification of the input data, (2) another model after classification of the input data depending on the size of the dam (small or large). The results indicated that the model using classification of the data gives estimates that reasonably match the observed breach widths more closely relative to the estimates of that without classification. The
MAE, $RMSE$, R^2 and E for entire data set without classification and for data after classification according to dam size are 15.52, 23.25, 0.80 and 0.80; 8.8, 16.3, 0.89 and 0.84, respectively, which are satisfactory in common model applications.
In order to assess the ability of the ANFIS model relative to that of the RE, the comparison was performed between their estimates using models having the same inputs and the same number of observations. The performances of ANFIS and RE in terms of the performance indices are presented

in Table 3. From the results given in Table 3, the ANFIS shows an improvement in predicting the average breach width.

Figure 7 present the ANFIS estimates compared to the actual observations for the scenarios using (h_w, S), (h_w, S, W) and $(h_w, S, W, C\ or\ NoC)$ as input variables, respectively. These ANFIS models were constructed using the same number of observation data (total=56 observations) in both the training and testing phases. ANFIS models for the same scenarios are conducted after classification of the data depending on the size of the dam (small or large).

As seen in Table 4, the same statistical indices as before were calculated for all the scenarios. To limit the space in this paper, only the results of ANFIS models using all data without classification are presented. Obviously, the performance of ANFIS model number 3 which uses all the hydraulic parameters $(h_w, S, W, C\ or\ NoC)$ as input variables is better than those of other models. The R^2 and E values of scenario 3 are higher than those of other models and MAE and $RMSE$ values of scenario 3 are smaller than those of other models either for entire data set or testing data.

The minimum values of MAE and $RMSE$ indices and maximum values of R^2 and E show the potential of the ANFIS methodology as a predictive tool for estimating the average breach width.

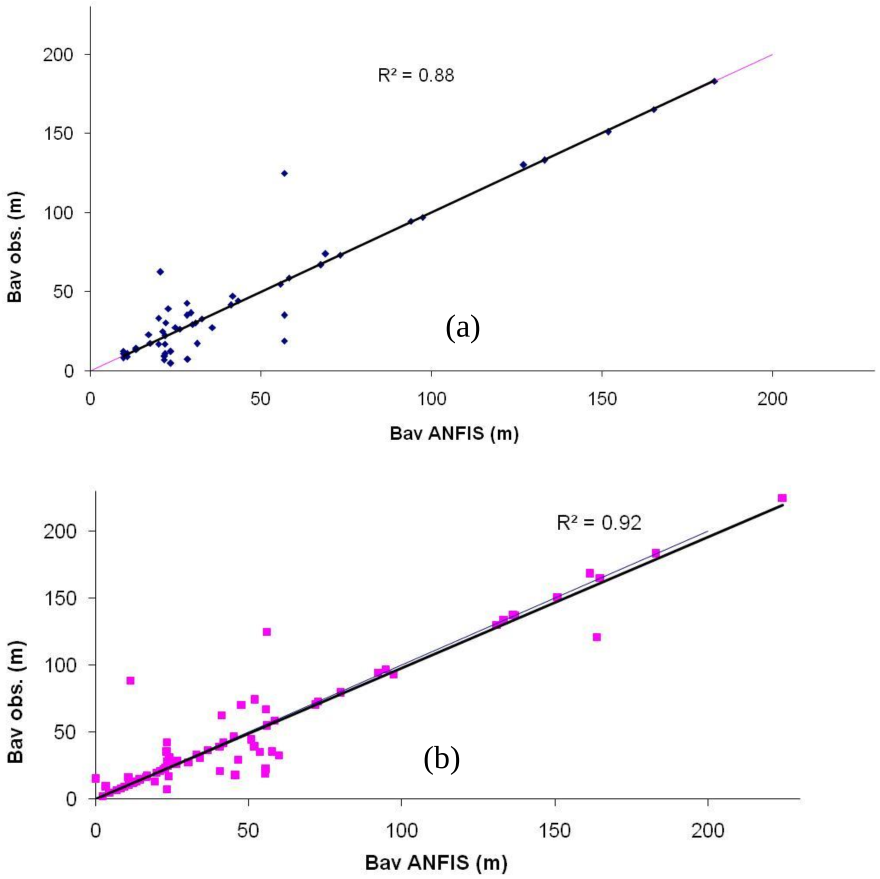


Fig. 6 Comparison between ANFIS estimates and observed data: a) model using all data without classification and b) model using data classification based on dam size

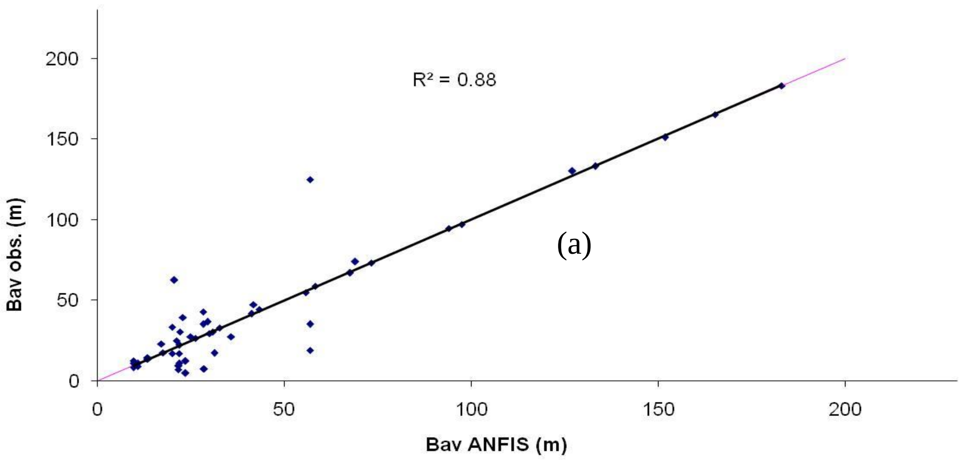
Table 3 Error criteria using the estimates of the ANFIS model and RE

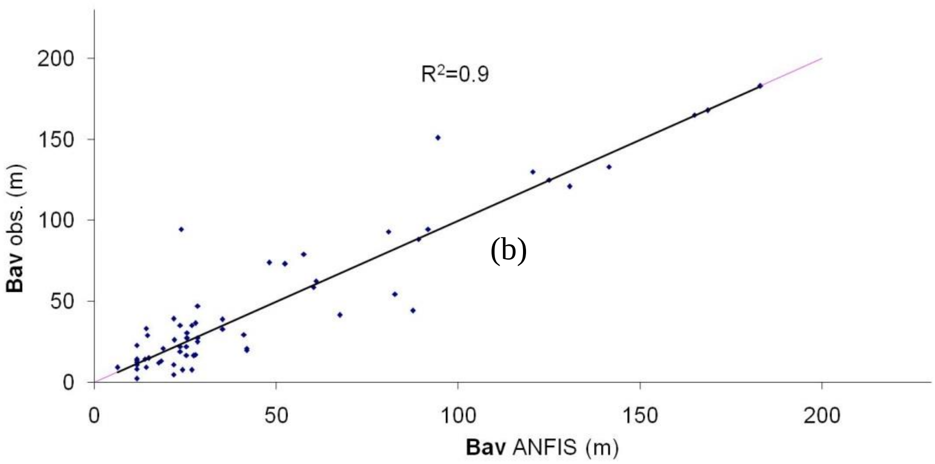
Inputs to ANFIS are h_w, S
Using data from 79 case studies

	without classification	with classification		Froehlich (2008)
		Small	Large	
MAE	15.52	5.98	11.66	14.0
$RMSE$	23.25	12.59	20.13	16.5
R^2	0.80	0.93	0.85	0.70
E	0.80	0.82	0.85	0.67

Table 4 Error criteria using the estimates of ANFIS models with different inputs

	Using data from 56 case studies		
	Inputs to ANFIS are		
	h_w, S	h_w, S, W	h_w, S, W, C or NoC
MAE	8.03	10.9	5.07
$RMSE$	14.8.0	17.0	13.0
R^2	0.88	0.85	0.92
E	0.89	0.90	0.91





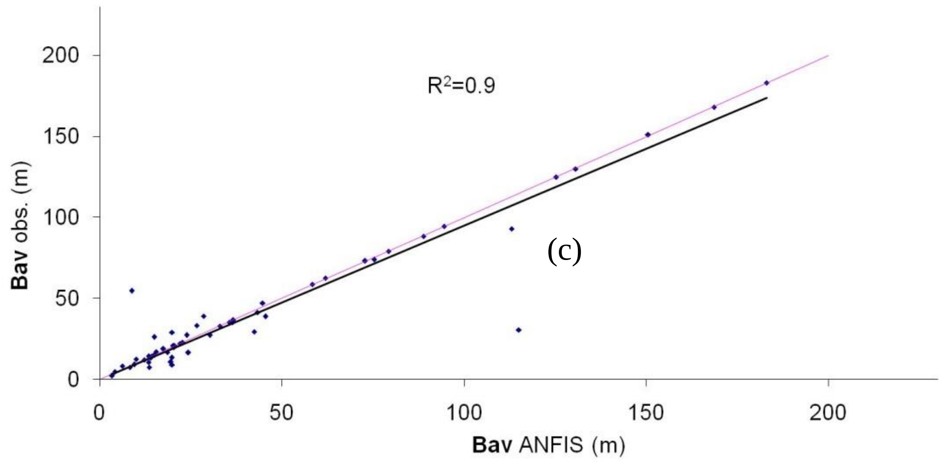


Fig. 7 Comparison between ANFIS estimates and observed data:
a) $B_{av} = f(h_w, S)$, b) $B_{av} = f(h_w, S, W)$ and c) $B_{av} = f(h_w, S, W, C\ or\ NoC)$

5. CONCLUSSIONS

Applicability of ANFIS method for predicting the average breach width of embankment dams was investigated. Models with different hydraulic parameters were constructed, trained and tested by the ANFIS method. Comparing the performance of ANFIS and RE models, the ANFIS model with triangular or Gaussian membership function and using all the hydraulic parameters $(h_w, S, W, C\ or\ NoC)$ as input variables had better performance and was selected as the best fitting model. moreover, the ANFIS model using classified data depending on the dam size and using (h_w, S) as input variables performs quite similar as that using all the hydraulic parameters. Those ANFIS models performed better than RE model and can estimate more accurate breach widths. The minimum values of MAE and $RMSE$ indices and maximum values of R^2 and E show the potential of the ANFIS methodology as a predictive tool for estimating the average breach width. the combination of linguistic rules of fuzzy logic with the training algorithm used in neural networks, contribute in more qualitative prediction results.

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