IMPLICIT NUMERICAL SCHEME FOR REGULATING UNSTEADY FLOW IN OPEN CHANNEL

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ABSTRACT

Inaccurate operation and management of irrigation canals often cause inequitable and inefficient water distribution. The adjustment of hydraulic structures along irrigation canals causes unsteady flow in the canals at the initiation of revised operation till it attains a new steady state. Unsteady flow in open channels can be classified into two types: routing and operation-type problems. In the routing problems, The Saint Venant equations are solved to get the discharge and water level in the future time series. Also, they are used in the operation problem to compute the inflow at the upstream section of the channel according to the prescribed downstream flow hydrographs. This type of problem is known as the inverse computation of open channel flow and can be used to compute a schedule of operation for regulating structures of the delivery system to get a predefined water demand at the downstream end of the channel. An inverse implicit finite difference scheme is presented herein based on the Preissmann scheme. The computation proceeds from the final time level towards the first time level. The computation results obtained using the inverse implicit scheme showed more stability and less oscillation than that obtained using the backward explicit scheme. The computed upstream discharge hydrograph, when used as upstream boundary condition in the implicit routing scheme, produced downstream discharge hydrograph very close to the required demand hydrograph.

INTRODUCTION

Unsteady flow in open channels is governed by the fully dynamic Saint Venant equation, which express the principles of conservation of mass and momentum. Mathematical models of unsteady flow in open channels are usually built for engineering purposes such as flood forecasting and defense, navigation, dam break analysis, and operation of irrigation and power canals.

Precise control of water in irrigation system is becoming very important due to the increasing of water demand. Any error in the upstream inflow can cause losses of water or shortage of water. The objective of operation along irrigation canals aims to compute a schedule of operation for regulating structures to maintain a required water demand precisely. Mathematical model based on Saint Venant equations were developed to predict the upstream inflow according to the required downstream flow hydrograph. This type of problem is known as operation problem or the inverse computation of open channel flow[2,5,6,8,9,11,12,13].

The finite difference methods that use a fixed set of locations (nodes) along the stream channel are divided into two subclasses: explicit methods and implicit methods [1,3,4,7,10]. These methods offer practical advantages in that the locations of solution values in space and time are fixed. Thus, analysis and presentation of the results is simplified. In the explicit difference methods, the dependent variables, at a rectangular grid point on an advanced time line are determined from the known values and conditions at grid points on the present time line or present and previous time lines. In the implicit methods, unsteady flow may be simulated at a subsequent time levels on the *x-t* plane by setting up as many equations as there are unknown dependent variables and solving them simultaneously using appropriate time boundary conditions.

The stability of implicit methods allows use of large time steps in the solution. The time step may still be limited in terms of accuracy of results, but the time step can be adjusted to simulate varying flow conditions and not be restricted by the Courant condition [1,3,4,7]. Implicit methods make it possible to simulate long time periods economically, with acceptable accuracy. An implicit method called the Preissmann four-point scheme has been used extensively to solve the saint Venant equations.

Two finite difference algorithms have been recently used for solving the inverse problem. The first one is the implicit [2] method with a boundary value problem rotated 90^o in the distance-time grid and the other is the explicit backward operation scheme presented in Liu, et al. 1992 [6]. In the implicit method, the specified depths and discharges at the downstream end of the channel are used as the initial conditions and the boundary conditions can be either a discharge profile or water depth profile at both the initial and the final computational time. The backward operation scheme is an explicit solution based on the Preissmann scheme. In this scheme, the computation begins at the top-right corner of the time-distance grid, then the solution is obtained cell by cell, moving first backward in space and then backward in time. Also, the solution of the backward explicit scheme can be obtained by proceeding first back ward in time and then backward in space [12].

An inverse implicit finite difference scheme is presented in this paper based on the Preissmann scheme to predict the upstream inflow according to the required downstream flow hydrograph. The results obtained using the inverse implicit scheme showed more stability and less oscillation than that obtained using the backward explicit scheme. The computed upstream discharge hydrograph, when used as upstream boundary condition in the implicit routing scheme, produced downstream discharge hydrograph very close to the required demand hydrograph.

GOVERNING EQUATIONS

Unsteady flow in open channels is mathematically governed by the continuity equation and the momentum equation. These equations commonly known as the Saint Venant equations. The Saint Venant equations can be formulated in different ways, depending on the assumptions used in their derivations. Assuming no lateral outflow, these equations can be written as:

$$\frac{\partial y}{\partial t} + \frac{1}{b}\frac{\partial Q}{\partial x} = 0 \qquad (1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\,A\left(\frac{\partial y}{\partial x} + S_f - S_0\right) = 0 \qquad (2)$$

where: A = wetted cross-sectional area; b = wetted top width; g = gravitational acceleration; Q = discharge (through A); y = depth of flow ; t = time; x = space; S_0= bottom slope of the channel and S_f = friction slope.

PREISSMANN IMPLICIT ROUTING MODEL

The implicit methods of finite difference were developed because of the large number of time steps required by an explicit schemes. Implicit method which can use a large time steps without any stability problems are generally preferred [1,3,4,7,10]. In this method of flow simulation, the basic set of partial differential equations is transformed into the corresponding finite difference equations. Transient flows may be obtained at subsequent time levels on the *x-t* plane by setting up many equations as there are unknown dependent variables and by solving them simultaneously using appropriate time boundary conditions.

The Preissmann scheme is the most widely applied implicit finite difference method because of its simple structure with both flow and geometrical variable in each grid point [1,3,4,7]. This implies a simple treatment of boundary conditions and a simple incorporation of structure and bifurcation points. Also, it has the advantages that steep wave fronts may be properly simulated by varying the weighting coefficient.

The computational grid for the Preissmann implicit scheme is shown in Figure 1. Using The application of the Preissmann implicit scheme to the system of equations (1) and (2) is written as follow:

$$\frac{\partial f}{\partial t} = \phi\frac{f_{j+1}^{n+1} - f_{j+1}^{n}}{\Delta t} + (1-\phi)\frac{f_j^{n+1} - f_j^{n}}{\Delta t} \qquad (3)$$

$$\frac{\partial f}{\partial x} = \theta\,\frac{f_{j+1}^{n+1} - f_j^{n+1}}{\Delta x} + (1-\theta)\,\frac{f_{j+1}^{n} - f_j^{n}}{\Delta x} \qquad (4)$$

$$f(x,t) = \theta\left[\phi\, f_{j+1}^{n+1} + (1-\phi)\, f_j^{n+1}\right] + (1-\theta)\left[\phi\, f_{j+1}^{n} + (1-\phi)\, f_j^{n}\right] \qquad (5)$$

where $f_j^n = f(j\Delta x, n\Delta t)$; Δx =space interval; Δt =time interval; ϕ = a weighting coefficient for distributing terms in space and θ = a weighting coefficient for distributing terms in time, 0≤θ≤1. All the variables with subscripts' *n* in the above expressions are known and all the variables with subscripts' *n+1* are the unknowns. Applying these numerical approximations to equations (1) and (2) yields:

$$C\ y_j^{n+1} + D\ Q_j^{n+1} + E\ y_{j+1}^{n+1} + F\ Q_{j+1}^{n+1} + G = 0 \qquad (6)$$

$$C'\ y_j^{n+1} + D'\ Q_j^{n+1} + E'\ y_{j+1}^{n+1} + F'\ Q_{j+1}^{n+1} + G' = 0 \qquad (7)$$

Where *C*, *D*, *E*, *F*, *G*, *C'*, *D'*, *E'*, *F'*, and *G'* are coefficients computed with known values at time level *n*. Equations (6) and (7) constitute a system of linear algebraic equations in four unknowns. As there are *J* points on the row *n+1*, there are *J-1* rectangular grids *and J-1* cells in the channel. Thus there are *2(J-1)* equations for the evaluation of *2J* unknowns. Two boundary conditions provide the necessary two additional equations to close the system. The double-sweep method [4,7,10] can be applied to get the solution of the linear algebraic equations.

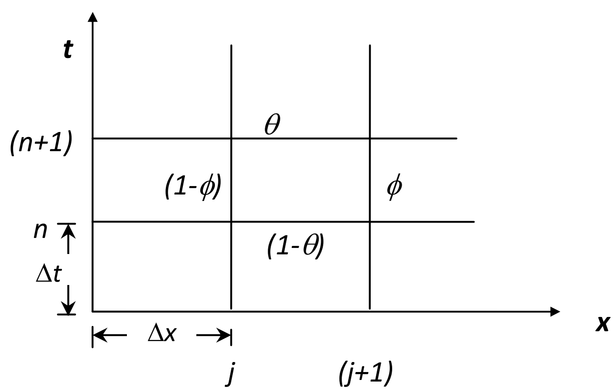

Figure. 1. Computational Grid for Preissmann Scheme

BACKWARD EXPLICIT SCHEME

The backward explicit scheme is an explicit solution based on the Preissmann scheme. The required discharge and water level downstream boundary conditions are specified in this scheme [6]. Considering the time level (*N*) as the final condition, Figure 2, and knowing Q_J , and y_J between any two time levels at the downstream section, the discharge and water depth profile at the time level (*N-1*) can be computed. The solution begins at the top-right corner of the time-distance grid. The application of the finite difference equations yields an algebraic system of two equations and two unknowns. The solution obtained cell by cell, moving first backward in space and then backward in time.

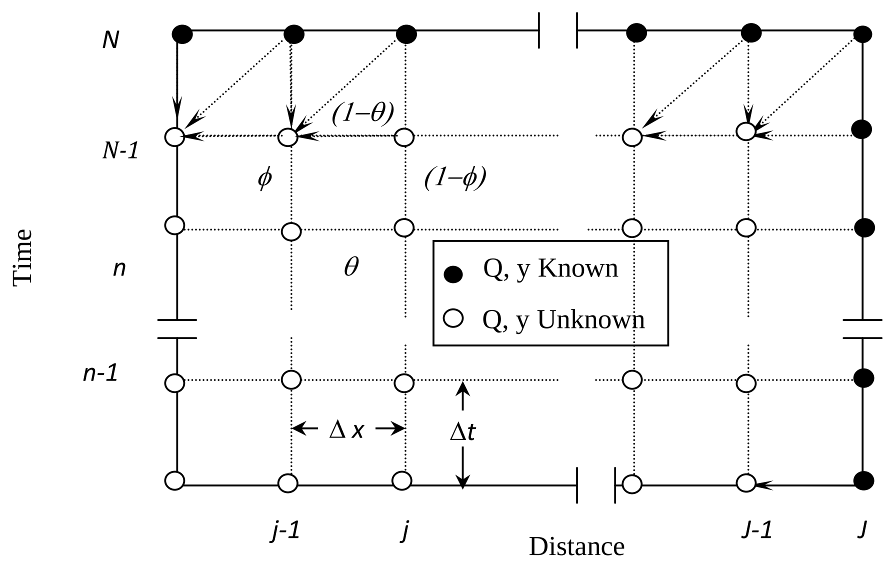

Figure. 2. Inverse Explicit Computational Grid

INVERSE IMPLICIT SCHEME

The inverse implicit scheme is an implicit solution based on the Preissmann scheme. The specified depths and discharges located at the final computational time are used as the initial condition in this method, Figure 3. The upstream boundary condition can be either the specified discharge profile or the specified water depth profile at the downstream end of the canal. The downstream boundary condition is the relationship between the flow depth and discharge at the beginning of the canal. In this inverse method, the Preissmann scheme is written as:

$$\frac{\partial f}{\partial t} = \phi \frac{f_{j-1}^{n} - f_{j-1}^{n-1}}{\Delta t} + (1-\phi)\frac{f_{j}^{n} - f_{j}^{n-1}}{\Delta t} \qquad (8)$$

$$\frac{\partial f}{\partial x} = \theta \frac{f_{j}^{n-1} - f_{j-1}^{n-1}}{\Delta x} + (1-\theta)\frac{f_{j}^{n} - f_{j-1}^{n}}{\Delta x} \qquad (9)$$

$$f(x,t) = \theta\left[\phi\, f_{j-1}^{n-1} + (1-\phi)\, f_{j}^{n-1}\right] + (1-\theta)\left[\phi\, f_{j-1}^{n} + (1-\phi)\, f_{j}^{n}\right] \qquad (10)$$

The solution proceeds towards the first time level by solving *2(j-1)* equation for the evaluation of *2j* unknowns at each time level. Thus, two more equations are needed for a unique solution. Two boundary conditions provide the necessary two additional equations to close the system.

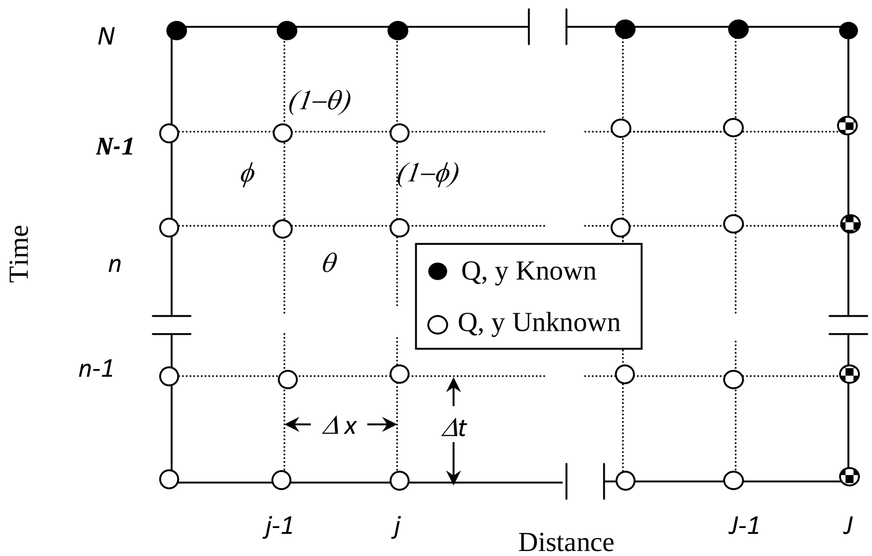

Figure. 3. Inverse Implicit Computational Grid

DESCRIPTION OF TEST CANAL

The performance of the inverse implicit scheme was tested using a trapezoidal channel with a bottom width of *5.0m* and side slopes *1.5H to 1V*. The bottom slope is *0.001*, Manning's $n = 0.025$, the channel length is *2.5 km*. At the downstream outlet, the discharge increases from $5\ m^3/sec$ to $10\ m^3/sec$ in one hour, it remains constant at $10\ m^3/sec$ for the next two hours, then decreases to $5.0\ m^3/sec$ in one hour (demand line in Figure 4.). Constant flow depth and discharge of steady uniform flow at the final computational time are specified as the initial condition. Discharge-water depth rating curve was used as the downstream boundary condition at the beginning of the channel. The discharge at the upstream intake, Figure 4, was computed using the specified discharge at the downstream end section as upstream boundary condition.

The obtained upstream discharge hydrograph was then used as upstream boundary condition and the discharge-water depth rating curve was used as the downstream boundary condition, to simulate the flow in the channel with the routing implicit finite difference scheme. The computed downstream hydrographs reasonably reproduced the prescribed demand, Figure 4.

NUMERICAL TESTS AND COMPARISON OF RESULTS

The computed upstream discharge hydrographs using both the backward explicit scheme and the inverse implicit scheme are shown in Figure 4. The space interval $\Delta x = 250\ m$ and time interval $\Delta t = 300\ sec$ were used in both the backward explicit method and the inverse implicit method. The weighting coefficient $\phi = 0.5$ and the weighting coefficient $\theta = 1.0$ were used in the inverse explicit method to damp the oscillations of the computed hydrographs. The weighting coefficient $\phi = 0.67$ and the weighting coefficient θ = *0.67* were used in the inverse implicit method. The computed results using the inverse implicit scheme showed approximately the same accuracy as the corresponding results obtained by the backward explicit scheme, Figure 4. The upstream discharge hydrograph obtained by the inverse implicit scheme, when used as upstream boundary conditions in the routing problem, reproduced approximately the same downstream discharge hydrograph as the corresponding hydrograph obtained using the backward explicit scheme. Also, the reproduced computed downstream discharge hydrographs were close to what are expected, Figure 4.

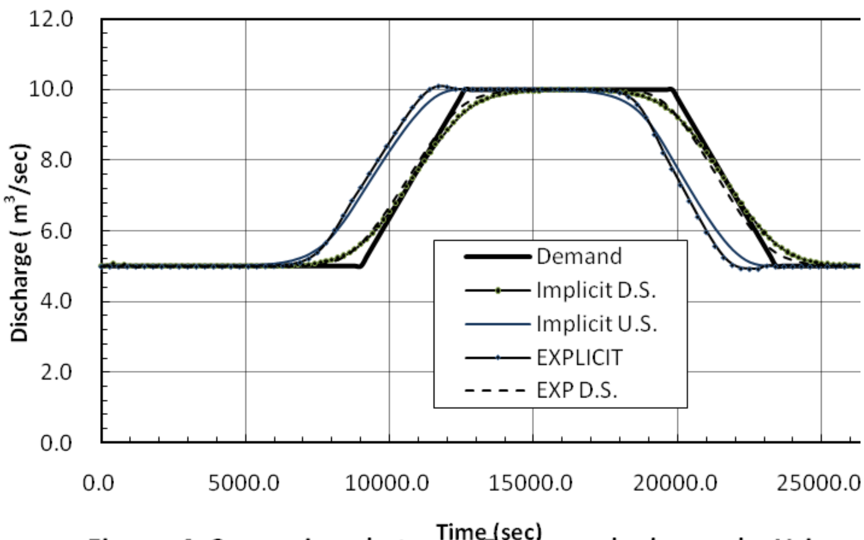

Figure. 4. Comparison between Discharge hydrographs Using Explicit Backward Operation Method and Inverse Implicit Method.

The inverse implicit scheme was tested with numerical experiments using the following parameters: the computation space interval Δx, the time interval Δt, the weighting coefficient ϕ , and the weighting coefficient θ. All these parameters were being constant except one of them which could be tested in sequence. The space interval Δx =*250 m*, the time interval Δt =*300 sec*, the weighting coefficient ϕ = *0.67*, and the weighting coefficient θ = *0.67* were kept constant in the computation of upstream hydrographs, except for the tested parameter.

The computed upstream discharge hydrographs using Δx =*100 m, 250 m*, and 50*0 m* are illustrated in Figure 5. As shown in Figure 5, the effect of the space interval on the computed upstream hydrographs has a negligible influence. Also, the computed upstream discharge hydrographs, using Δt =*300 sec, 600 sec,* and *900 sec* showed approximately the same results as illustrated in Figure 6.

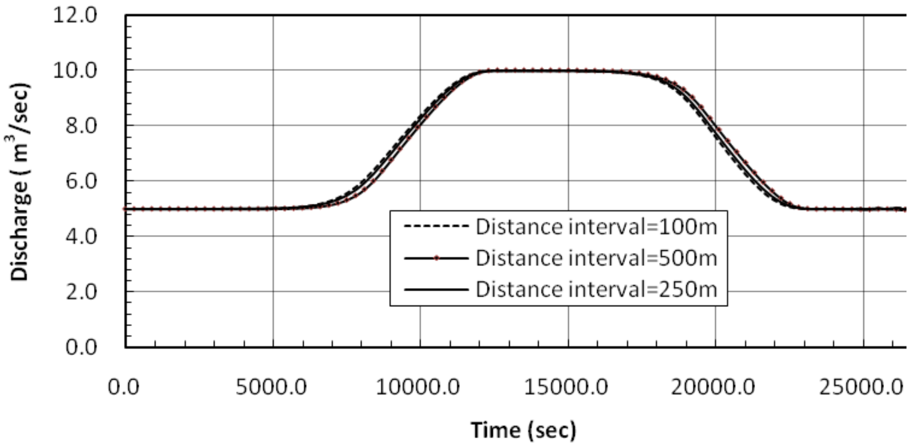

Figure 5. Computed Upstream Discharge Hydrographs Using Inverse Implicit Scheme at Different Distance Intervals

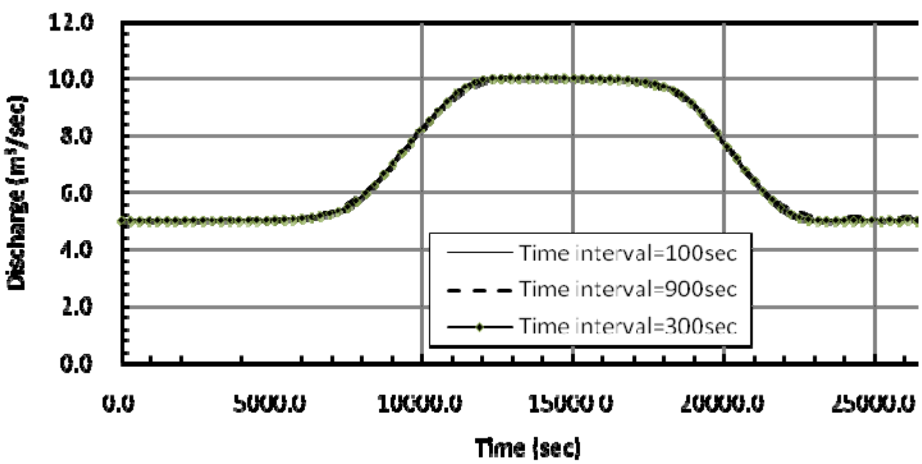

Figure 6. Computed Upstream Discharge Hydrographs Using Inverse Implicit Scheme at Different Time Intervals

The computed upstream discharge hydrographs using the inverse implicit scheme with weighting coefficient $\theta = 0.67$, and $\theta = 1.0$ showed approximately the same accuracy as illustrated in Figure 7, while the results obtained using the backward explicit scheme showed that the stability is reduced when θ decreases from *1* to *0.68* as illustrated in Figure 8.

The computed upstream discharge hydrographs using the inverse implicit scheme with weighting coefficient $\phi = 0.67$, and $\phi = 0.9$ gave approximately the same results as shown in Figure 9, while the results obtained using the backward explicit scheme showed that the stability is reduced when ϕ increases from *0.5* to *0.8* as illustrated in Figure 10.

The results obtained using the inverse implicit scheme with different values of weighting coefficient θ and ϕ showed more accuracy and less oscillation than that obtained using the backward explicit scheme.

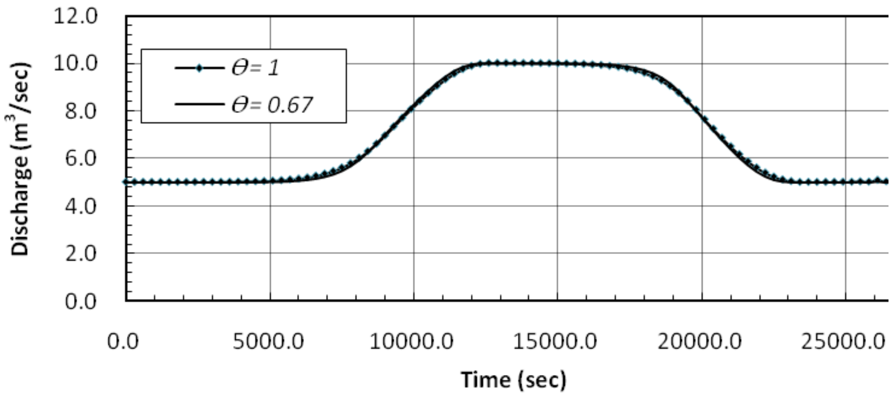

Figure 7. Computed Upstream Discharge Hydrographs Using Inverse Implicit Scheme with Different Θ factors

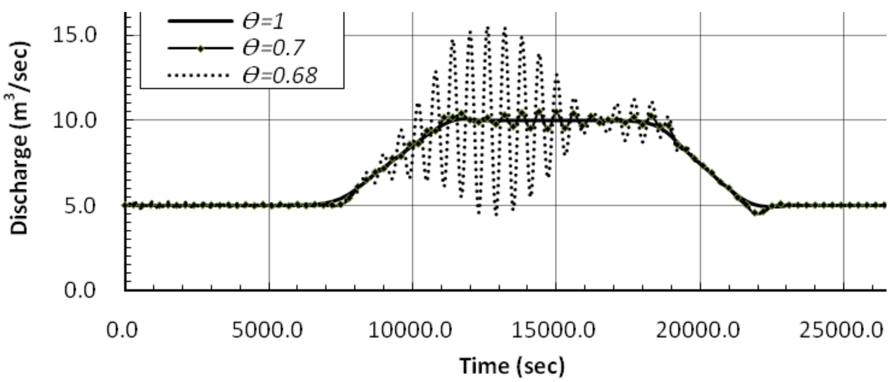

Figure 8. Computed Upstream Discharge Hydrographs Using Backward Explicit Scheme with Different Θ factors

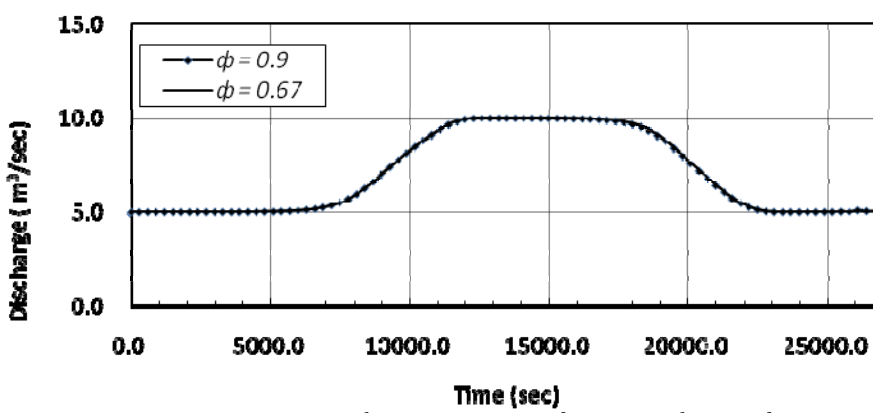

Figure 9. Computed Upstream Discharge Hydrographs Using Inverse Implicit Scheme with Different Φ factors

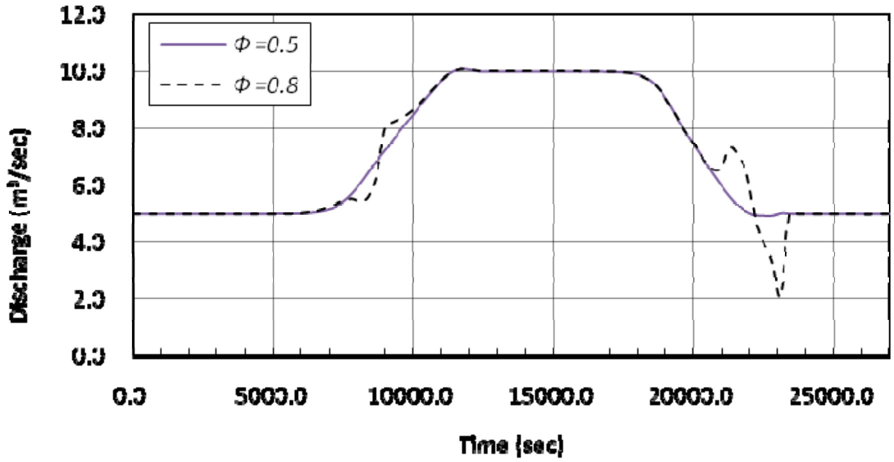

Figure 10. Computed Upstream Discharge Hydrographs Using Backward Explicit Scheme with Different Φ factors

CONCLUSIONS

A finite difference algorithm for regulating unsteady flow in open channel has been presented. The method is implicit and numerically stable. It can be used to compute the upstream inflow and setting of controlling structures according to the required downstream outflow. This inverse implicit scheme was tested with the following parameters: the space interval Δx, the time interval Δt, the weighting coefficient ϕ, and the weighting coefficient θ.

The effect of the computational space interval on the computed upstream hydrographs and the reproduced downstream hydrographs could be neglected for the used values of Δx. Also, the computed results obtained using the inverse implicit scheme with different time intervals gave approximately the same results.

The computed upstream discharge hydrographs using the inverse implicit scheme with different weighting coefficient θ showed approximately the same accuracy, while the results obtained using the backward explicit scheme showed that the stability is reduced when θ decreases from *1* to *0.68*.

In the inverse implicit scheme, the effect of the weighting coefficient ϕ on the computed upstream discharge hydrographs is negligible, while the results obtained using the backward explicit scheme showed that the stability is reduced when ϕ increases from *0.5* to *0.8*. The backward explicit scheme is unstable for values of ϕ bigger than *0.8*.

The inverse implicit scheme is stable for all the tested factors, while the backward explicit scheme is stable only with appropriate weighting coefficient. The computed results using the backward explicit scheme with appropriate weighting coefficients (θ

= *1.0*, ϕ =0.5) showed the same accuracy as that obtained using the inverse implicit scheme . All the inverse solutions, when used as upstream boundary condition to the routing model, reproduced downstream discharge hydrographs very close to the required outflow hydrographs.

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NOTATION

The following symbols are used in this paper:

A = wetted cross-sectional area;
b = wetted top width;
f = general function;
g = gravitational constant;
j = cross-section index;
n = time-level index;
Q = discharge (through A);
S_0 = bottom slope of the channel;
S_f = friction slope;
t = time;
x = space;
y = depth of flow ;
Δt = time interval;
Δx = space interval;
ϕ = a weighting coefficient for distributing terms in space; and
θ = a weighting coefficient for distributing terms in time.