

THERMOPHORESIS PARTICLE DEPOSITION ON FLAT SURFACES DUE TO FLUID FLOW IN DARCY-FORCHHEIMER POROUS MEDIUM

Rebhi A. Damseh¹ and Kamel Alzboon²

¹ Mechanical Engineering Department, Al-Husn University College
Al-Balqa Applied University, P.O. Box 50, Irbid, Jordan
Email: rdamseh@yahoo.com

² Environmental Engineering Department, Al-Husn University College
Al-Balqa Applied University, P.O. Box 50, Irbid, Jordan

ABSTRACT

An analysis is performed for non-Darcy free convection flow over a vertical plate embedded in a thermally stratified, fluid saturated porous medium and taking into account the presence of thermophoresis particle deposition. A finite-difference scheme was used to solve the system of transformed governing equations. Numerical results for the details of the velocity, temperature and concentration profiles which are shown on graphs have been presented. It is shown that the inertia forces have a significant influence on the flow characteristics in this problem. Comparison with previous published work is performed and the results are found to be in excellent agreement.

Keywords: Non-Darcy, Porous Medium, Thermophoresis, Natural Convection, Flat Plate.

INTRODUCTION

Thermophoresis is a phenomenon, which causes small particles to be driven away from a hot surface and toward a cold one. Small particles, such as dust, when suspended in a gas temperature gradient, experience a force in the direction opposite to the temperature gradient.

Thermophoresis is of practical importance in many engineering applications when hot gases containing small suspended particles flow over cool surfaces. For example, thermophoresis can be effective in removing or collecting small particles from laminar gas streams in air leaning and aerosol sampling devices [1]. There are several other practical situations where we come across this phenomenon as an origin for the deposition of particulate matter on surfaces of heat exchangers, causing scale formation with the attendant reduction of the heat-transfer coefficient [2]. In certain applications such as the microelectronics industry, deposition of contaminant particles by thermophoresis on wafers in clean rooms during manufacturing steps can be a major

cause of loss of product yield [3]. Also, thermophoretic deposition of radio active particles is considered to be one of the important factors causing accidents situation in nuclear reactors [4].

Goren [5] studied the role of thermophoresis of a viscous and incompressible fluid. The classical problem of flow over a flat plate is used to calculate deposition rates and it is found that the substantial changes in surface deposition can be obtained by increasing the difference between the surface and free stream temperatures. Gokoglu and Rosner [6] and Park and Rosner [7] obtained a set of similarity solutions for the two dimensional laminar boundary layers and stagnation point flows respectively. Chiou [8] obtained the similarity solutions for the problem of a continuously moving surface in a stationary incompressible fluid, including the combined effects of convection, diffusion, wall velocity and thermophoresis. Grag and Jayaraj [9] discussed the thermophoresis of small particles in forced convection laminar flow over inclined plates; Epstein et al. [10] have studied the thermophoresis transport of small particles through a free convection boundary layer adjacent to a cold, vertical deposition surface in a viscous and incompressible fluid.

Flow, heat and mass transfer in porous media has been studied extensively in recent years. This is due to the increasing need in understanding the complicated transport process for application of diverse fields which include geothermal engineering, building insulation, energy conservation, solid matrix heat exchangers, filtration processes, underground disposal of nuclear waste materials, and many more. These problems are well documented in the books by Nield and Bejan [11] and Pop and Ingham [12]. Early studies on flow through porous medium were based on the Darcy law. The thermophoresis effect in Darcian porous medium is investigated by Chamkha and Pop [13]. It is generally recognized that the Darcy model is valid under the condition of low velocities. However, it is well known that Darcy model is valid when the Reynolds number based on the pore size is less than unity. For high velocity flow situations and/or porous material of large pore radius, Forchheimer modification is introduced by adding the quadratic inertial term to the original Darcy model. More recent studies can be found in Al-Hussien et al. [14] and Al-Odat et al. [15, 16].

This work is extension to the discussed model by Chamkha and Pop [13], so consideration is given to the influence of the inertia forces on fluid flow with the presence of thermophoresis effects on free convection heat and mass transfer problems from vertical surfaces embedded in porous medium.

PROBLEM FORMULATION

Consider the two-dimensional, non-Darcy, free convection boundary layer flow of a viscous incompressible fluid past an isothermal vertical plate of constant temperature T_w and concentration C_w . The ambient temperature is T_∞ and concentration C_∞ . The plate temperature T_w and concentration C_w are higher than the ambient temperature T_∞ and

concentration C_∞ . It is assumed that the fluid properties are constant except the influence of density variation with temperature is considered only in the body force term. The flow is assumed to be in the x-direction, which is along the vertical plate in the upward direction, and y-axis is taken to be normal to the plate, Fig. 1. Allowing for both Brownian motion of particles and thermophoretic transport, the governing equations are, Chamkha and Pop [13], Chiou [8]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial y} + \frac{F\sqrt{K}}{\nu} \frac{\partial u^2}{\partial y} = \frac{K\beta_T g}{\nu} \frac{\partial T}{\partial y} + \frac{K\beta_C g}{\nu} \frac{\partial C}{\partial y} \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + \frac{\partial(Cv_t)}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \tag{4}$$

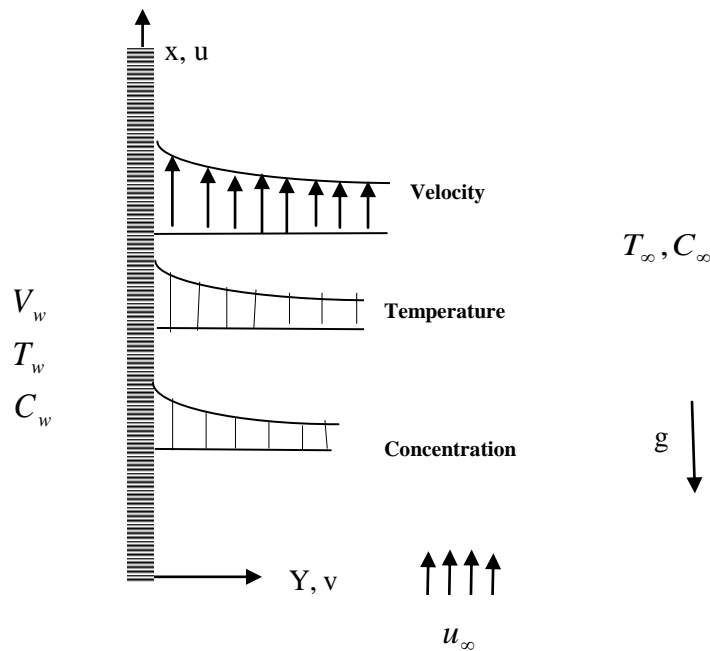


Fig. 1. Schematic diagram for flow model and coordinate system

The physical problem assumes the following boundary conditions:

$$\begin{aligned} y=0 & \quad u=0 & \quad v=0 & \quad T(0,t)=T_w & \quad C(0,t)=C_w \\ y \rightarrow \infty & \quad u=0 & & \quad T=T_\infty & \quad C=C_\infty \end{aligned} \quad (5)$$

Here x and y are the dimensional distance along and normal to the plate, respectively. (u , v) are the averaged velocity components along the x and y , directions respectively, T is the temperature, C is the concentration, β_T and β_C are the coefficient of thermal expansion of temperature and concentration respectively. ν is the kinematic viscosity, α_m is the effective thermal diffusivity, K is the permeability of the medium, F is the inertia coefficient and D is the Brownian diffusion coefficient.

The thermophoretic velocity v_t can be expressed in the from,

$$v_t = -k \frac{\nu}{T} \frac{\partial T}{\partial Y} \quad (6)$$

where k is the thermophoretic coefficient.

In order to non-dimensionalize the governing equations, we introduce the following non-dimensional parameters:

$$\begin{aligned} X = \frac{x}{l}, \quad U = \frac{u}{u_o}, \quad V = Ra^{1/2} \frac{v}{u_o}, \quad Y = Ra^{1/2} y/l, \quad V_t = Ra^{1/2} v_t / u_o, \\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \psi = \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \quad (7)$$

where $u_o = gK \beta_T (T_w - T_\infty) / \nu$ is the characteristic velocity, $Ra = gK \beta_T (T_w - T_\infty) / \nu \alpha$ is the Rayleigh number, l is the characteristic length of the plate. The dimensionless form of the governing equations and their boundary conditions are reduced to:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (8)$$

$$\frac{\partial U}{\partial Y} + \Gamma \frac{\partial U^2}{\partial Y} = \frac{\partial \theta}{\partial Y} + N \frac{\partial \psi}{\partial Y} \quad (9)$$

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial Y^2} \quad (10)$$

$$U \frac{\partial \psi}{\partial X} + V \frac{\partial \psi}{\partial Y} + \frac{\partial(\psi + V_t)}{\partial Y} = \frac{1}{Le} \frac{\partial^2 \psi}{\partial Y^2} \quad (11)$$

$$V_t = -k \frac{Pr}{N_t + \theta} \frac{\partial \theta}{\partial Y} \quad (12)$$

$$\begin{aligned} Y = 0 & \quad V = 0 & \quad \theta = 1 & \quad \psi = 1 \\ Y \rightarrow \infty & \quad U = 0 & \quad \theta = 0 & \quad \psi = 0 \end{aligned} \quad (13)$$

where $\Gamma = FKg\beta_T \sqrt{K}(T_w - T_\infty)/\nu^2$ is the dimensionless inertia coefficient expressing the relative importance of the inertia effect, $Pr = \nu/\alpha_m$ and $Le = \alpha_m/D$ are the Prandtl and Lewis numbers respectively, $N_t = (T_w - T_\infty)/T_\infty$ is the thermophoresis parameter and $N = \beta_C(C_w - C_\infty)/\beta_T(T_w - T_\infty)$ is the buoyancy parameter.

Equations (8-13) can be transformed from the (X, Y) coordinates to the dimensionless coordinate (ξ, η) by introducing the following non-dimensional variables:

$$\xi = X, \quad \eta = Y\xi^{-1}, \quad \psi = f(\eta)\xi^{-1}, \quad \theta = \theta(\eta), \quad \psi = \psi(\eta) \quad (14)$$

In the equations above, the stream function ψ satisfied the continuity equation (8) with $U = \partial\psi/\partial Y$ and $V = -\partial\psi/\partial X$. Finally one can obtain the following system of dimensionless equations:

$$f'' + 2\Gamma f f'' = \theta' + N\psi' \quad (15)$$

$$\theta'' + \frac{1}{2} f \theta' = 0 \quad (16)$$

$$\frac{1}{Le} \psi'' + \frac{1}{2} f \psi' + \frac{k Pr}{N_t + \theta} \left[\theta' \psi' + \psi \theta'' - \frac{\psi}{N_t + \theta} \theta'^2 \right] = 0 \quad (17)$$

$$\begin{aligned} f(0) = 0, \quad \theta(0) = 1, \quad \psi(0) = 1 \\ f' \rightarrow 0, \quad \theta \rightarrow 0, \quad \psi \rightarrow 0 \quad \text{as } \eta \rightarrow \infty \end{aligned} \quad (18)$$

The quantities of physical interest are the wall themophoretic deposition velocity V_{tw} and the local Sherwood number that can be expressed as:

$$V_{tw} = -k \frac{Pr}{N_t + 1} \theta'(0) \quad (19)$$

$$Sh_x = -\sqrt{Ra_x} \psi'(\xi, 0) \quad (20)$$

RESULTS AND DISCUSSION

For this present problem, numerical computations have been carried out. It is clearly seen that the results are given values of the parameters N (buoyancy ratio), k (thermophoresis coefficient), N_t (temperature ratio) and Γ (inertia parameter).

The resulting ordinary differential Equations (15-17) under boundary conditions (18) have been solved by means of the fourth-order Runge-Kutta method with symmetric estimation of $f'(0)$, $\theta'(0)$, and $\psi'(0)$ by the shooting technique. The basic step size used for the calculation is $\Delta\eta = 5 \times 10^{-3}$. This value was arrived at after performing many numerical experiments to access grid independence. An iteration process is employed and continued until the desired results are obtained within the following convergence criterion:

$$\left| \frac{j_{i+1} - j_i}{j_{i+1}} \right| \leq 10^{-6} \quad (21)$$

where j stands for $f, \theta, \text{ or } \psi$ and i refers to space coordinate.

Furthermore, the accuracy of the numerical results obtained in this investigation was validated by a direct comparison with the solutions reported by Chamkha and Pop [13] and Bakier and Mansour [17]. Table 1 presents a comparison between the values of $-\theta'(0)$ and $-\psi'(0)$ for $\Gamma = 0$ obtained by the present numerical method with those corresponding to the above mentioned references. It is clearly seen from Table 1 that excellent agreement between the respective results is obtained.

Table 1 Values of $-\theta'(0)$ and $-\psi'(0)$ for $\Gamma = 0$ compared to those obtained by Chamkha and Pop (2004) and Bakier and Mansour (2007).

	$k = 0, N = 0, Le = 1$			$k = 0, N = Le = 1$		
	Chamkha and Pop	Bakier and Mansour	Present study	Chamkha and Pop	Bakier and Mansour	Present study
$-\theta'(0)$	0.44325	0.4438	0.4432	0.62783	0.6276	0.6274
$-\psi'(0)$	--		--	0.62783	0.6276	0.6274

Figures 2, 3, and 4 show the effect of inertia parameter Γ on velocity, temperature and concentration profiles at different values of buoyancy ratio N and for values of $N_t = 10.0$, $k = 0.4$, $Le = 5.0$, and $Pr = 0.72$. The Figures show that as the buoyancy parameter is increased the velocity is increased due to favorable slip velocities near vertical surfaces and concentration contribution in immigration of fluid particles from the vertical surfaces. On the other hand, it is clearly shown from the figures that the velocity in the non-Darcy model is dramatically decreased, while the temperature and concentration in the non-Darcy model is increased as the inertia parameter increased. This behavior

attributed to the fact that increasing the inertia parameter has a tendency to resist the flow. Furthermore, when temperature and concentration ratios are increased; this is due to small temperature differences between vertical surface and free stream conditions, a decrease in heat and mass transfer process is occurred as shown in Figure 5.

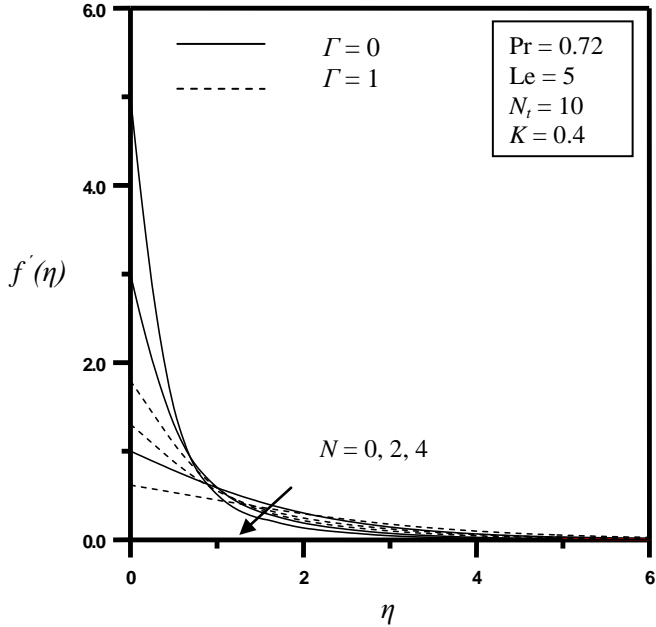


Fig. 2. Effect of inertia parameter Γ on the velocity distribution at different values of buoyancy ratio N

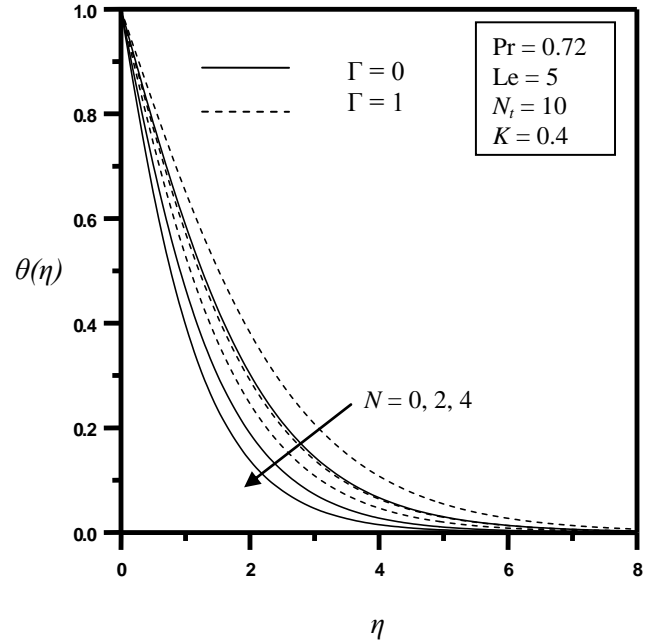


Fig. 3. Effect of inertia parameter Γ on the temperature distribution at different values of buoyancy ratio N

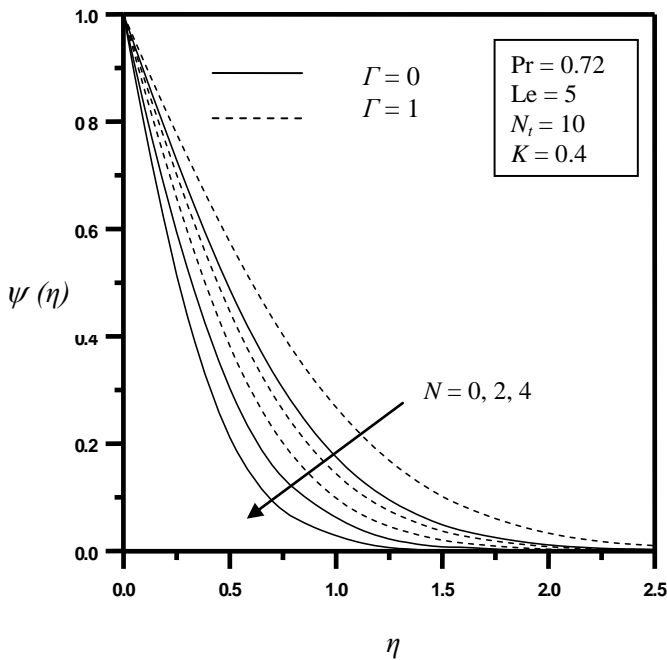


Fig. 4. Effect of inertia parameter Γ on the concentration distribution at different values of buoyancy ratio N

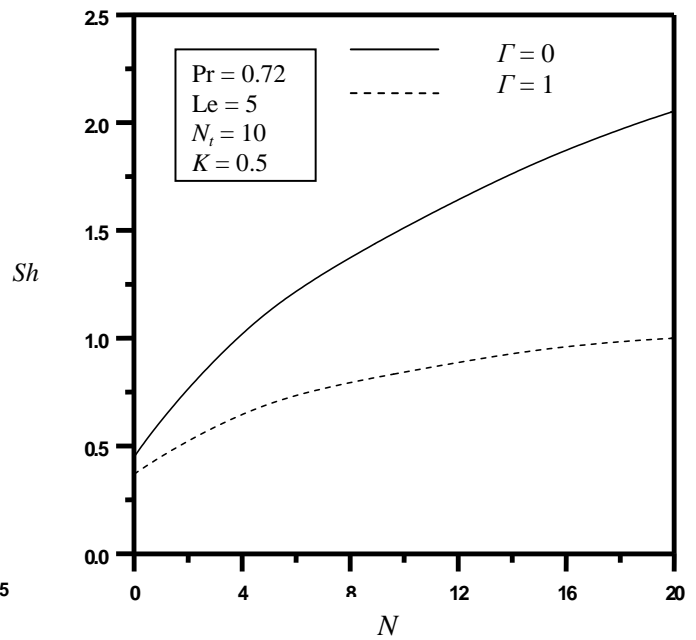


Fig. 5. Effect of buoyancy ratio on the Sherwood number for both Darcy and non-Darcy models

Figure 6 represents the dependent thermophoretic deposition velocity V_{tw} on buoyancy ratio N for both Darcy and non-Darcy models. The other parameters are assigned values of $Le = 5.0$, and $Pr = 0.72$. It is clear from the figure, and as said before, the inertia force tends to slow down the flow. This in turn decreases the deposition velocity at the wall. On the other hand, the decrease in temperature difference between vertical surface and free stream conditions plays, due to dependency of deposition velocity on temperature gradient at the wall, a big role in decreasing the wall deposition velocity. The figure also shows that as the thermophoresis coefficient k and buoyancy parameter increase the wall thermophoresis velocity is also increased; this is due to favourable temperature gradients and concentration velocities contribution in immigration of fluid particles from the vertical surfaces. Clearly, the thermophoresis values are decreased when temperature ratios are increased; this is due to small temperature differences between vertical surface and free stream conditions.

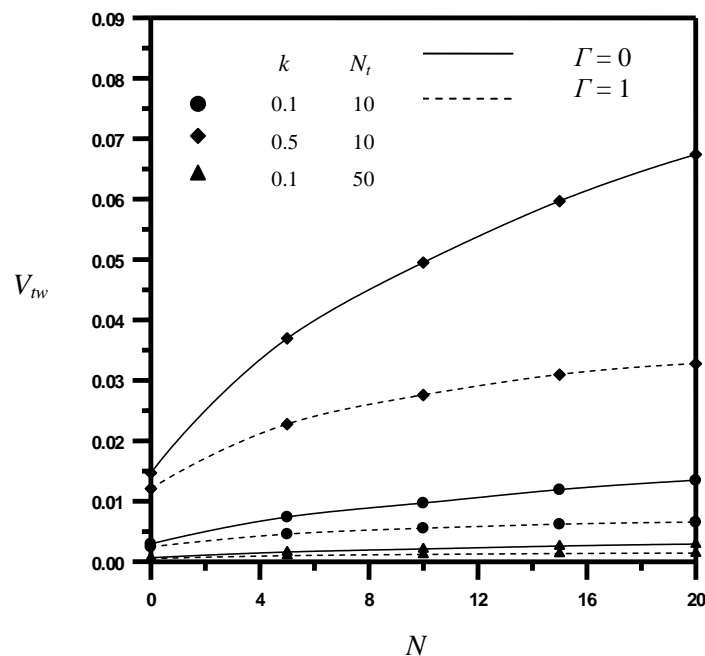


Fig. 6. Effect of buoyancy ratio on the wall deposition velocity for both Darcy and non-Darcy models

CONCLUSIONS

The effect of inertia force on heat and mass transfer free convection problem of a Newtonian fluid over a flat vertical plate embedded in non-Darcy medium in the presence of thermophoresis particle deposition effect were studied. In this problem, the non-Darcy model which contributes for the inertia force is employed to describe the flow in porous medium. It was found that the inclusion of the inertia parameter in the calculation can cause a slight increase in the fluid temperature and concentration, in

addition to a significant decrease in the fluid velocity. In turns, the deposition velocity at the wall will decrease. The thermophoretic deposition velocity increased as the thermophoresis constant k and the buoyancy parameter N increased and as the temperature ration N_t decreased.

NOMENCLATURE

C	Fluid concentration
c_p	Specific heat capacity
D	Brownian diffusion coefficient
F	Dimensional inertia coefficient
g	Gravitational acceleration
K	Permeability of the porous medium
k	Thermophoresis coefficient
Le	Lewis number
N	Buoyancy ratio, $[\beta_C (C_w - C_\infty) / \beta_T (T_w - T_\infty)]$
N_t	Dimensionless temperature ratio, $T_\infty / [T_w - T_\infty]$
Pr	Prandtl number
Ra	Local Rayleigh number, $Kg\beta(T_w - T_\infty)x / \nu\alpha$
Sh	Sherwood number
T	Temperature
u, v	Velocity components in x - and y -directions
v_t	Thermophoresis velocity
v_{tw}	Thermophoresis velocity at wall
V_t	Dimensionless thermophoresis velocity
V_{tw}	Dimensionless thermophoresis velocity at wall
x, y	Axial and normal coordinates

Greek symbols:

α	Effective thermal diffusivity of the porous medium
β_T	Coefficient of thermal expansion, $(-1/\rho)(\partial\rho/\partial T)_p$
β_C	Coefficient of concentration expansion, $(-1/\rho)(\partial\rho/\partial C)_p$
η	Non-dimensional transformed variable
Γ	Dimensionless inertia parameter
θ	Dimensionless temperature
ψ	Dimensionless concentration
ν	Effective kinematic viscosity
ρ	Fluid density

Subscripts

w	Surface conditions
∞	Free stream condition
t	Thermophoresis effects

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