EXPLORING THE VERSATILITY OF THE IMPLICIT METHOD OF CHARACTERISTIC (MOC) FOR TRANSIENT SIMULATION OF PIPELINE SYSTEMS

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ABSTRACT

In this paper, versatility of the Implicit Method of Characteristics is further explored. This method is proposed as a versatile method capable of simulating any pipeline system with an arbitrary combination of devices in the system. The method was proposed as a remedy to the limitation of the mostly used conventional Method of Characteristics (MOC). In this method, an element-wise definition is used for all the devices that may be used in a pipeline system and the corresponding equations are derived in an element-wise manner. The proper equations defining the behavior of each device including pipes are then assembled to form the system of equations to be solved for the unknown nodal heads and flows. This method is applied here to an example problem of transient flow caused by failure of a pump with and without a check valve and results are presented and compared with those of explicit MOC. The results show that the implicit MOC can used for any combination of devices to accurately predict the variations of head and flow in the pipeline system.

Keywords: water hammer, transient flow, method of characteristics, pipeline system, implicit method, boundary conditions

1. INTRODUCTION

Water hammer is produced by a rapid change of the flow velocity in the pipelines that may be caused by sudden valve opening or closure, starting or stopping the pumps, mechanical failure of an item, rapid changes in demand condition, etc. It usually result in violent changes of the pressure head, which is then propagated in the pipeline in a form of a fast pressure wave leading to sever damages. Design and operation of any pipeline system requires that the distribution of head and flow in the system predicted at different operating conditions. Many researchers have attempted the simulation of transient flow in pipes with different methods.

Chaudhry and Hossaini [1] solved the water hammer equations by MacCormak, Lambda and Gabutti explicit Finite Difference (FD) [1] schemes. They found that second order FD schemes result in better solutions than the first order MOC. Filion and Karney [2] proposed a method that combines a numerical integration method with a transient simulation model to improve the accuracy and capabilities of extended-period simulations in pipe networks. Their method analyzes a water distribution system for short time periods near the start and end of a time step using a transient model, and then uses the intuitions to predict the behavior of the system with a modified improved Euler approach. Their method leads a significant increase in simulation accuracy, but requires more system information and computational effort. Zhao and Ghidaoui [3] formulated, applied, and analyzed first and second-order explicit finite volume (FV) Gadunov-type schemes [3] for water hammer problems. They compared the FV schemes and MOC schemes with space line interpolation for three test cases with and without friction. They modeled the wall friction using the formula of Brunone et al. (1991). They found that the first order FV Gadunov scheme produces the same results with MOC considering space line interpolation. They also showed that, for a given level of accuracy, the second-order Gadunov-type scheme requires much less memory storage and execution time than the first-order Gadunov-type scheme. Sibetheros et al [4] showed numerical analysis of water hammer in a frictionless horizontal pipe using the method of characteristic (MOC) requires interpolations with spline polynomials. Wood [5], [6] compared MOC and Wave characteristics Method (WCM). He showed that for the same modeling accuracy, the WCM would require fewer calculations and less execution time. In addition, he showed that the number of calculations per time step required by WCM does not increase when more accuracy is required while for the MOC, the number of calculations per time step is roughly proportional to the accuracy. Ghidaoui and Kolyshkin [7] performed linear stability analysis of base flow velocity profiles for laminar and turbulent water hammer flows. They found that the main parameters that govern the stability behavior of the transient flows are the Reynolds numbers and the dimensionless time scale.

2. IMPLICIT MOC

The Implicit Method of Characteristics (IMOC) was proposed by Afshar and Rohani [8] as a remedy for the shortcomings of the conventional explicit MOC. In the conventional MOC, system devices such as valves, pumps, turbines, etc. are treated as boundary condition for the pipe segments. The method, therefore, is capable of handling pipeline systems in which only once device is located between any two pipes. Otherwise, a new boundary condition has to derived and used for any combination of the devices in the system. The Implicit Method of Characteristics is a method that can consider any arbitrary combination of devices in a pipeline system without requiring special treatment.

Here the main components of the implicit MOC as suggested by Afshar and Rohani [8] is briefly reviewed. In this method, an element-wise definition is used for all of the devices that may be used in a pipeline system and their governing equations are derived. The governing equations describing the behavior of each element are then assembled and subsequently solved for the unknown head and flow at discretized points. These equations are often written in a matrix form as follows:

$$S^e X^e = b^e \qquad (1)$$

In which X^e is the vector of element unknowns usually defined as $\left(\left[Q_i, Q_j, H_i, H_j\right]^{n+1}\right)^T$, S^e is the stiffness matrix and b^e is the right hand side vector.

In what follows, the stiffness matrix and right hand side vectors describing the transient behavior of different components of a pipeline system are presented. More detailed explanation of the method can be found elsewhere [8].

2.1 Pipe

For a pipe segment defined by two end points of *i* and *j*, the governing equation can be written in a matrix form as follows:

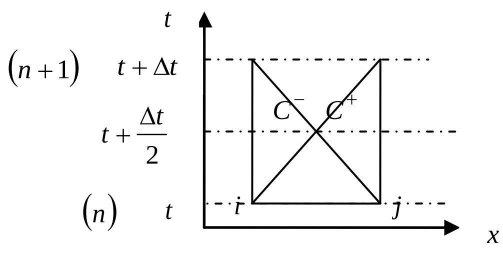

Fig. 1: Pipe element used in implicit MOC

$$S^e_{pipe} X^e_{pipe} = b^e_{pipe}$$

$$X^T = \left(\left[Q_i, Q_j, H_i, H_j\right]^{n+1}\right)^T \quad (2)$$

$$S^e_{pipe} = \begin{bmatrix} 0 & C & 0 & C_a \\ D & 0 & -C_a & 0 \end{bmatrix} \qquad b^e_{pipe} = \begin{bmatrix} E \\ F \end{bmatrix}$$

$$C = 1 + \frac{1}{2} R\Delta t Q^n_j Sign\left(Q^n_j + Q^n_i\right) + \frac{1}{2} R\Delta t Q^n_i Sign\left(Q^n_j + Q^n_i\right)$$

$$D = 1 + \frac{1}{2} R\Delta t Q^n_i Sign(Q^n_i + Q^n_j) + \frac{1}{2} R\Delta t Q^n_j Sign(Q^n_i + Q^n_j)$$

$$E = Q^n_i + C_a H^n_i - \frac{1}{4} R\Delta t\left(Q^n_i\right)^2 Sign\left(Q^n_j + Q^n_i\right) + \frac{1}{4} R\Delta t\left(Q^n_j\right)^2 Sign\left(Q^n_j + Q^n_i\right)$$

$$F = Q_j^n - C_a H_j^n - \frac{1}{4} R\Delta t \left(Q_j^n\right)^2 Sign\left(Q_i^n + Q_j^n\right) + \frac{1}{4} R\Delta t \left(Q_i^n\right)^2 Sign\left(Q_i^n + Q_j^n\right) \quad (3)$$

Where $R = \frac{f}{2DA}$ = term of friction factor, Q = discharge, H = pressure head, A = area of the pipe, a = velocity of the pressure wave, g =acceleration due to gravity, t = time, f = friction factor of the pipe, D = diameter of the pipe, x = distance along the pipe, indexes *n,n,+1* refer to the values at the previous and current time step and $C_a = \frac{gA}{a}$.

2.2 Reservoir

Now consider a reservoir considered as a hydraulic element shown in Fig. 2. The hydraulic behavior of this element can be defined via the head loss equation due to orifice and the condition of fixed head at the reservoir. These equations can be defined with the following element matrix and right hand side vector:

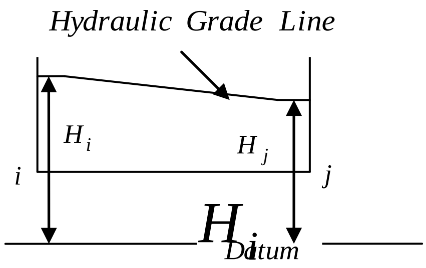

Fig. 2: Reservoir element used in implicit MOC

$$S_{res}^e = \begin{bmatrix} 1 & -1 & 0 & 0 \\ B & B & 1 & -1 \end{bmatrix} \quad b_{res}^e = \begin{bmatrix} 0 \\ C \end{bmatrix}$$

$$B = \frac{-2(1+k)}{8gA^2}\left(Q_i^n + Q_j^n\right) Sign\left(Q_i^n + Q_j^n\right) \quad (4)$$

$$C = \frac{-(1+k)}{8gA^2}\left(\left(Q_i^n\right)^2 + 2Q_i^n Q_j^n + \left(Q_j^n\right)^2\right) Sign\left(Q_i^n + Q_j^n\right)$$

Where k = the coefficient of entrance loss.

2.3 Check Valve

Now consider a check valve defined as a hydraulic element with two end points *i* and *j*. The function of the check valve can be defined with a junction or a dead end depending on the direction of the flow. Assuming a positive flow direction from *i* to *j*, the equations governing the behavior of the CV can defined with the following parameters if the flow is positive:

$$S_{cv}^{e} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ C & C & 1 & -1 \end{bmatrix} \qquad b_{cv}^{e} = \begin{bmatrix} 0 \\ D \end{bmatrix}$$

$$C = \frac{-2k}{8gA^2}\left(Q_i^n + Q_j^n\right) Sign\left(Q_i^n + Q_j^n\right)$$

$$D = \frac{-k}{8gA^2}\left(\left(Q_i^n\right)^2 + 2Q_i^n Q_j^n + \left(Q_j^n\right)^2\right) Sign\left(Q_i^n + Q_j^n\right) \tag{5}$$

Where k = the friction factor of the valve.

When the flow is reversed, the check valve is functional and its behavior can be modeled with the following stiffness matrix and right hand side vector to ensure no flow across the valve:

$$S_{cv}^{e} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix} \qquad b_{cv}^{e} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{6}$$

2.4 Pump

For a pump element shown in Fig. 3, the following stiffness and right hand side matrices can be defined:

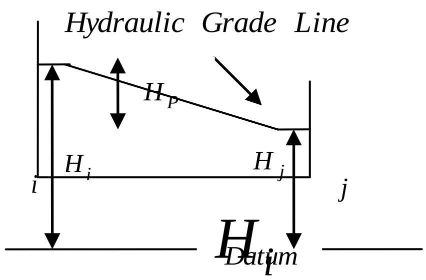

Fig. 3: pump element used in implicit MOC

$$S_{pump}^{e} = \begin{bmatrix} C & 0 & 1 & -1 \\ 0 & D & 1 & -1 \end{bmatrix} \qquad b_{pump}^{e} = \begin{bmatrix} E \\ F \end{bmatrix}$$

$$E = \frac{H_R}{n_P Q_R}\left[2a_1 \frac{Q_i^n}{n_P Q_R} - a_2 \alpha_P^{n+1} + 2a_2 \frac{Q_i^n}{n_P Q_R} \tan^{-1} \frac{\alpha_P^{n+1} n_P Q_R}{Q_i^n}\right]$$

$$Q_i^n - H_R\left(\left(\alpha_P^{n+1}\right)^2 + \frac{\left(Q_i^n\right)^2}{n_P^2 Q_R^2}\right)\left(a_1 + a_2 \tan^{-1} \frac{\alpha_P^{n+1} n_P Q_R}{Q_i^n}\right)$$

$$F = \frac{H_R}{n_P Q_R}\left[2a_1 \frac{Q_j^n}{n_P Q_R} - a_2 \alpha_P^{n+1} + 2a_2 \frac{Q_j^n}{n_P Q_R} \tan^{-1} \frac{\alpha_P^{n+1} n_P Q_R}{Q_j^n}\right]$$

$$Q_j^n - H_R\left(\left(\alpha_P^{n+1}\right)^2 + \frac{\left(Q_j^n\right)^2}{n_P^2 Q_R^2}\right)\left(a_1 + a_2 \tan^{-1} \frac{\alpha_P^{n+1} n_P Q_R}{Q_j^n}\right)$$

$$C = \frac{H_R}{n_P Q_R}[2a_1 \frac{Q_i^n}{n_P Q_R} - a_2 \alpha_P^{n+1} + 2a_2 \frac{Q_i^n}{n_P Q_R} \tan^{-1} \frac{\alpha_P^{n+1} n_P Q_R}{Q_i^n}]$$

$$D = \frac{H_R}{n_P Q_R}[2a_1 \frac{Q_j^n}{n_P Q_R} - a_2 \alpha_P^{n+1} + 2a_2 \frac{Q_j^n}{n_P Q_R} \tan^{-1} \frac{\alpha_P^{n+1} n_P Q_R}{Q_j^n}] \qquad (7)$$

$$\alpha_P^{n+1} - C_6\left(\alpha_P^{n+1}\right)^2\left(a_3 + a_4 \tan^{-1} \frac{\alpha_P^{n+1} n_P Q_R}{Q_i}\right) - C_6 \frac{Q_i^2}{n_P^2 Q_R^2}\left(a_3 + a_4 \tan^{-1} \frac{\alpha_P^{n+1} n_P Q_R}{Q_i}\right) = \alpha + C_6 \beta \qquad (8)$$

Where n_p = number of parallel pumps, index *R* related to the rated condition, a_1 and a_2 = constants for the straight lines representing the head characteristic curve, $C_6 = -\frac{15 T_R \Delta t}{\pi \; I \; N_R}$, $T_R = \frac{60\gamma \; H_R Q_R}{2\pi \; N_R \; \eta_R}$, γ = specific weight of liquid, η_R = pump efficiency at rated condition, *I* = combined polar moment of inertia of the pump, *N* = rotational speed of the pump in rad/s, α^n, β^n = the values at previous time step, a_3 and a_4 = constants for the straight lines representing the torque characteristic curve.

3. NUMERICAL APPLICATION

The flexibility and versatility of the implicit MOC is examined here with four numerical examples. First example considers the case of transient flow caused by a pump failure proposed by Chaudhry [9]. The pipeline system consists of two parallel pumps, two pipes, a frictionless junction, and a reservoir as shown in Fig. 4 with the following characteristics: Length of the first pipe $L = 450\ m$, first pipe diameter $D = 0.75\ m$, friction factor $f = 0.01$, wave velocity $a = 900\ m/s$; Length of the second pipe $L = 550\ m$, second pipe diameter $D = 0.75\ m$, friction factor $f = 0.012$, wave velocity $a = 1100\ m/s$. The pump data is as follows: $Q_R = 0.25\ m^3/s$, $H_R = 60\ m$, $N_R = 1100\ rpm$, $I = 16.85\ kg.m^2$ *per pump*. The characteristic data of the pumps can be found in Chaudhry [9]. The difference between water levels in upstream and downstream reservoirs is *59 m*. Figs.5 and 6 show the variation of the pump head and discharge during 15 seconds following the failure obtained by implicit MOC. This problem is also solved using the conventional MOC as formulated by Chaudhry [9]. So that a comparison can be made with the results obtained using the implicit method. Fig. 7 shows the relative error of the pump's head obtained with the implicit and explicit method. This figure indicates a good agreement between two methods.

The second example is similar to the first one except for a check valve which is now located downstream of the pumps as shown in Fig. 8. Analysis of this pipeline system using conventional MOC requires that the equations describing the check valve function are combined with those of pump to arrive at a new boundary condition for the first pipe shown in Fig. 8. With the implicit MOC, however, the system can be easily modeled by positioning a CV element downstream of the pump before the first pipe. The method, therefore, can be used for any pipeline system with arbitrary number of devices in the system. The variation of pump head and flow obtained using the implicit MOC is shown in Figs. 9 and 10. It is interesting to note that the system response to the pump failure is virtually the same for two cases of example one and two before the check valve is functional at time 2.7 seconds. Following the reversal of the flow, the CV stops the flow in the system as shown in Fig. 10.

The third example is again similar to the first example with the difference that the system now has two pumping stations of the first example in series, each station consisting of two parallel pumps. Figures 11 and 12 show the variation of pumping head and discharge with time obtained using the proposed method while Fig. 13 shows the relative error with respect to the conventional MOC. Once again, analysis of this system required a special treatment of the serial pumps in which the equations governing the transient behavior of each set of pumps was combined to arrive at the proper boundary condition. Analysis of this system using the implicit MOC required no special treatment except for considering two pump elements of the same characteristics in series.

The fourth example is similar to the third one in which a check valve is located downstream of the serial pumps. Analysis of this system using explicit MOC required that equations of serial pumps were combined with those of CV, leading to very complicated boundary condition. The system, however, was easily analyzed with the implicit MOC leading to the results shown in Figs. 14 and 15.

4. Conclusion

In this paper, the versatility and flexibility of the Implicit Method of Characteristics was investigated. This method was originally proposed as a remedy to the shortcomings of the conventional Method of Characteristics (MOC). An element-wise definition was used for the devices used in a pipeline system and the equations defining the behavior of each device including pipes are then assembled to form the system of equations to be solved for the unknown nodal heads and flows. The method was applied here to four problems of transient flow caused by failure of pumping system and results were presented and compared with those of explicit MOC, when available. The results show the ability of the proposed method to handle any configuration of devices in a pipeline system while producing accurate results.

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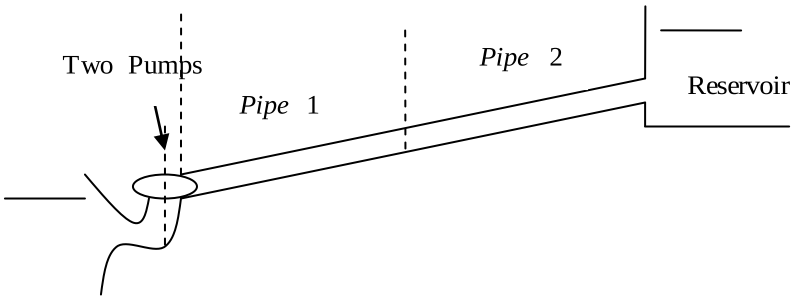

Fig. 4: Sketch of the piping system for the first example

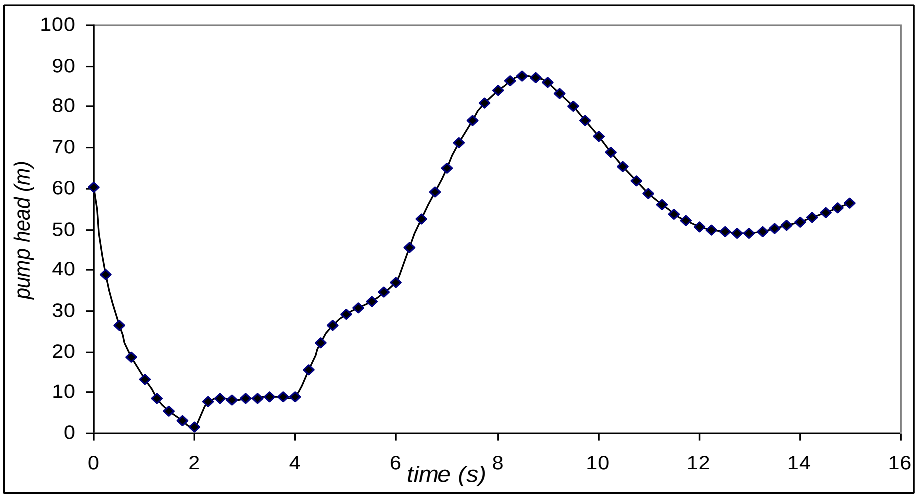

Fig. 5: Pressure head Vs time at pump by implicit MOC (first example)

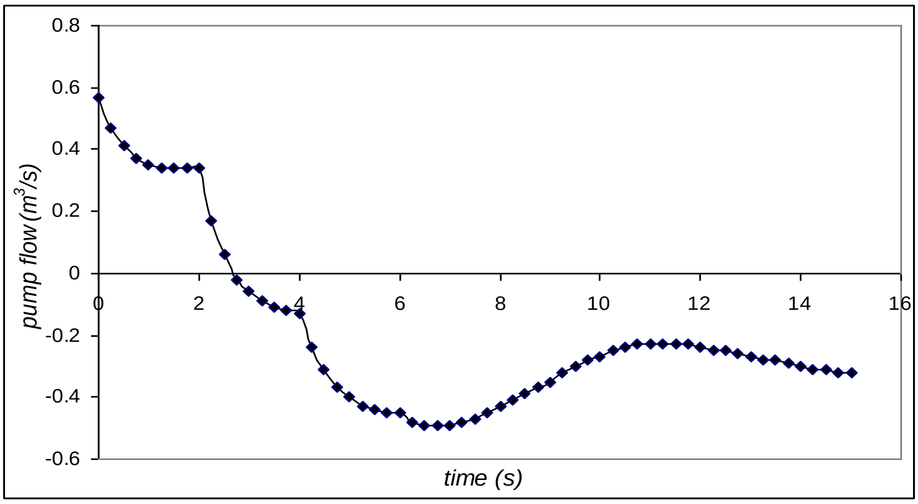

Fig. 6: Flow Vs time at pump by implicit MOC (first example)

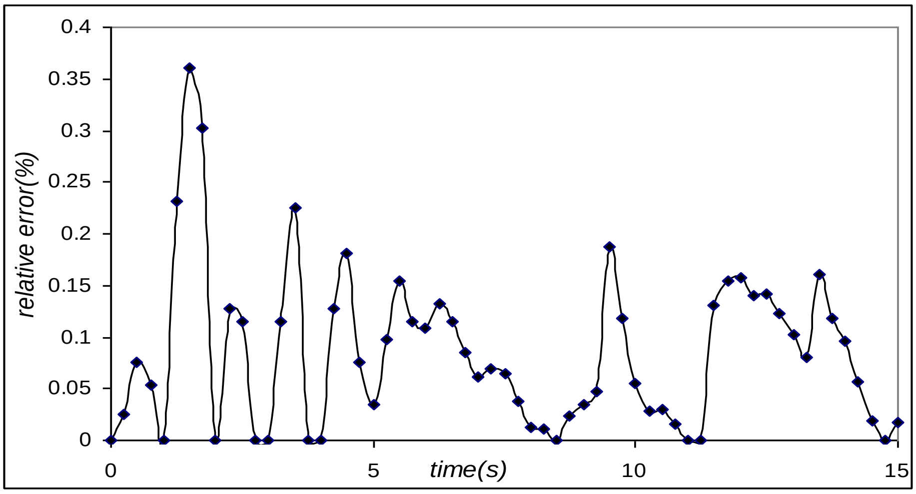

Fig. 7: Relative Error of explicit and implicit MOC vs. time (first example)

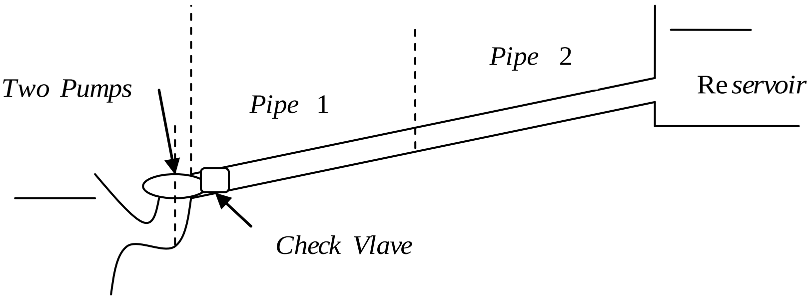

Fig. 8: Sketch of the piping system for the second example

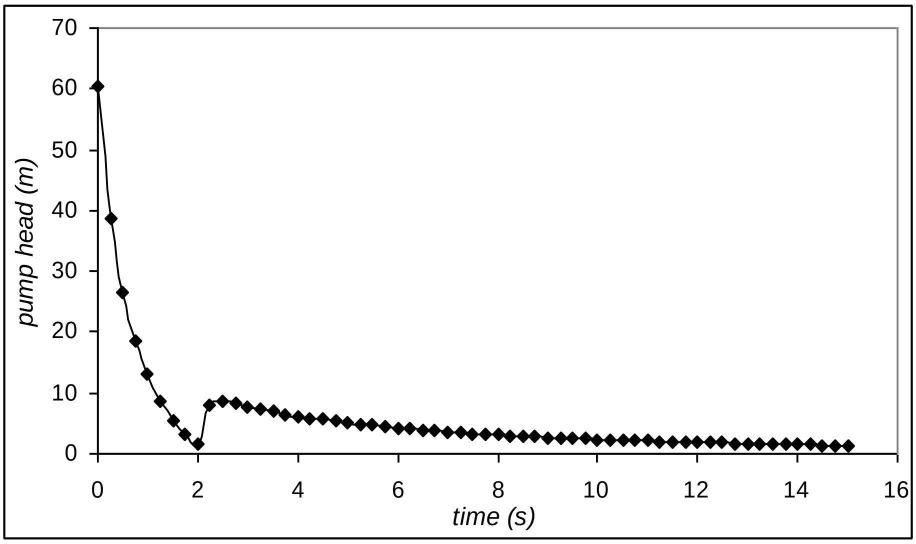

Fig. 9: Pressure head Vs time at pump by implicit MOC (second example)

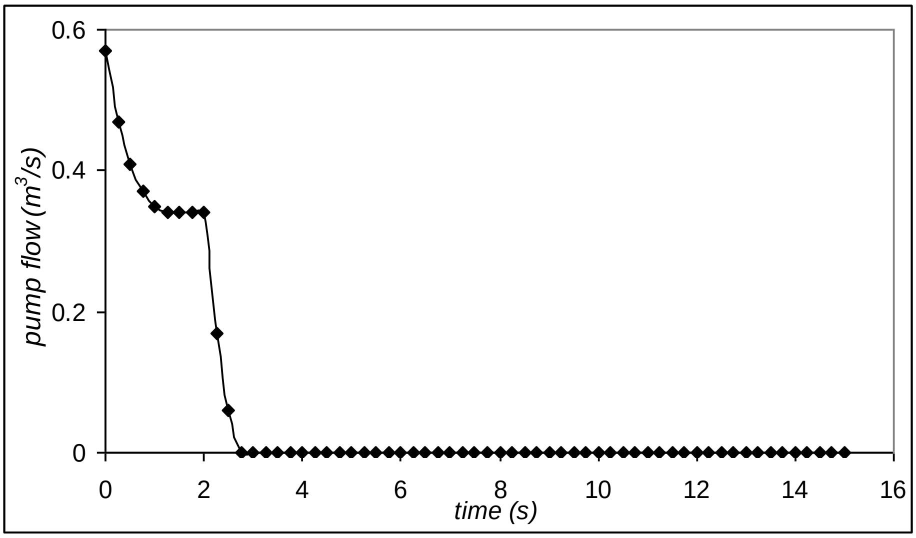

Fig. 10: Flow Vs time at pump by implicit MOC (second example)

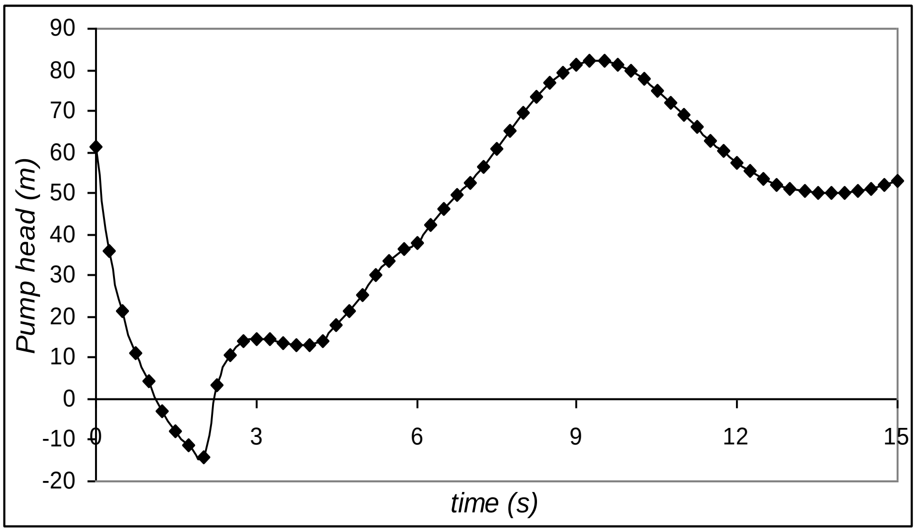

Fig. 11: Pressure head Vs time at pump by implicit MOC (third example)

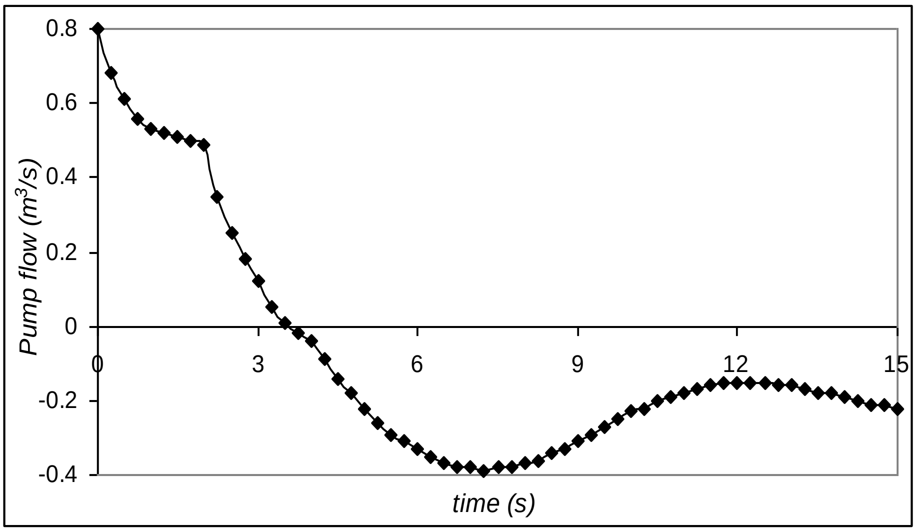

Fig. 12: Flow Vs time at pump by implicit MOC (third example)

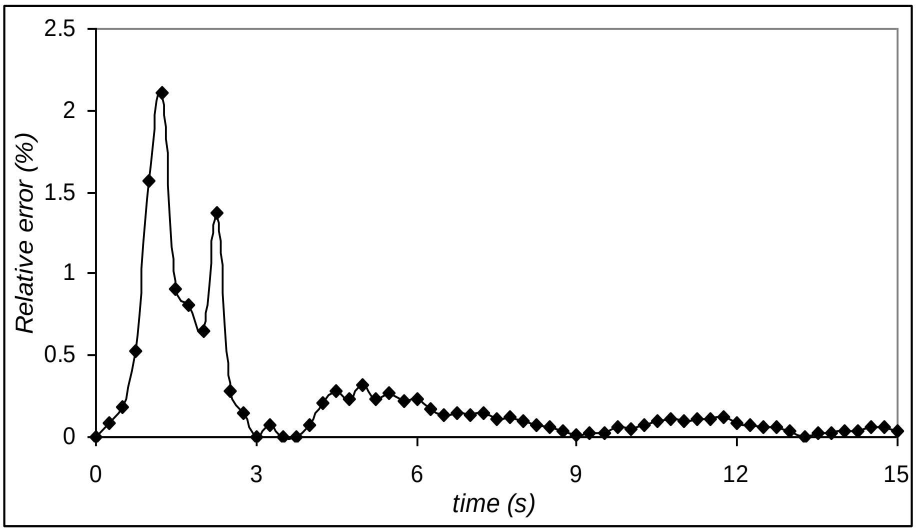

Fig. 13: Relative Error of explicit and implicit MOC vs. time (third example)

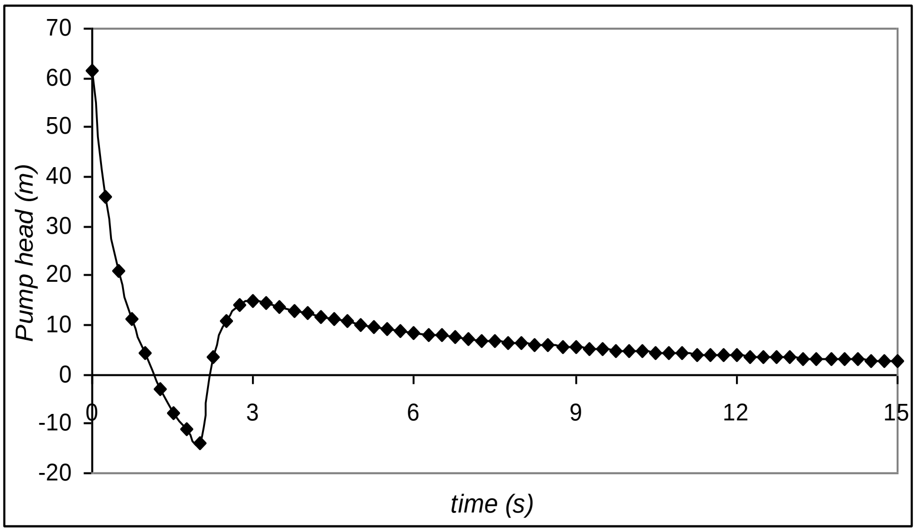

Fig. 14: Pressure head Vs time at pump by implicit MOC (fourth example)

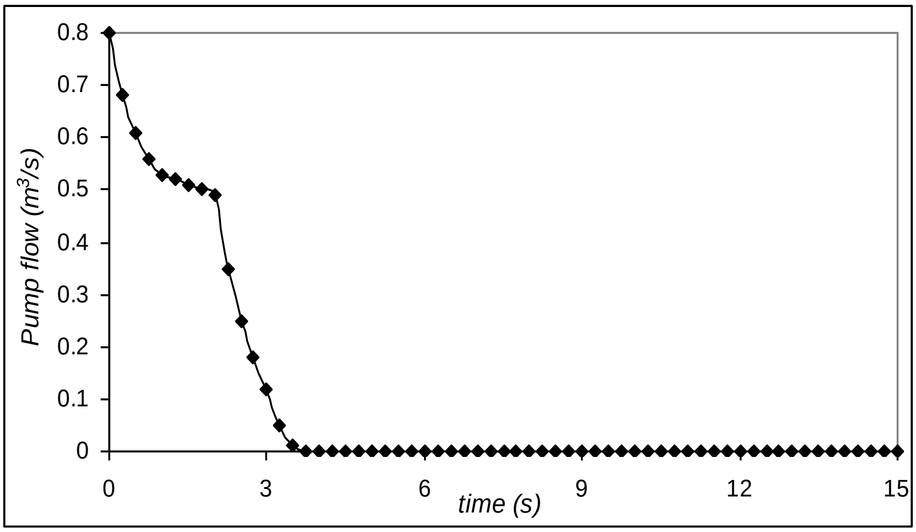

Fig. 15: Flow Vs time at pump by implicit MOC (fourth example)