A PROPOSED MODIFICATION TO THE WHITE, BETTESS AND PARIS RATIONAL REGIME APPROACH

M. A. Haidera

Senior Lecturer in Hydraulic Engineering,
Faculty of Engineering, Sana'a University, Yemen
Email: mhaidera@hotmail.com

ABSTRACT

The White, Bettess and Paris (WBP) rational regime approach for the design of straight stable trapezoidal channels presented the formulation which was used by HR Wallingford to produce tables for the design of stable alluvial channels. Small-scale laboratory tests of these methods persistently produce channels that are wider and shallower than predicted. To investigate this discrepancy, wide and narrow trapezoidal channels were studied in terms of wetted perimeter, hydraulic radius and side slope and compared with equal-area rectangular channels. It was found that the hydraulic radius, side slope and the threshold of motion are the main factors influencing the difference between the predicted and observed geometric shapes and sediment concentration. Based on this, a modification to the WBP method in terms of hydraulic radius and threshold of motion was suggested. Flume data were used to highlight the improvement made by this modification.

INTRODUCTION

Methods for determining equilibrium geometry of alluvial bankfull channels are grouped into empirical, mechanistic, and extremal hypothetical approaches. The empirical regime approach (e.g. Lacey [11]) relies on data collected from stable natural rivers and artificial channels and hence it is only applicable to situations similar to those for which the data were collected.

The mechanistic or tractive force approach (Diplas[10]) employs the basic laws of mechanics to obtain expressions for predicting the equilibrium geometry of alluvial channels below threshold of motion. The extremal hypothesis is an analytical approach, which employs a variational principle as part of its formulation. This variational principle has been frequently expressed in terms of the maximisation or minimisation of a parameter, such as minimisation of energy or slope (Chang [9]), minimisation of Froude number (Yalin[17], Yalin & Da Silva [18]), or maximisation of sediment transport (White et al. [16]). Equations for sediment transport, flow resistance and continuity are combined with this principle as a fourth necessary relationship to determine channel width and to predict regime or equilibrium channel condition. The extremal hypothesis based on the maximisation of sediment transport (White et al. [15] & [16]) is used more frequently as it provides general, if not exact, agreement with a wide range of observations.

OBJECTIVES AND PROCEDURE

The aim of this study was to extend the database of straight, bankfull, equilibrium laboratory channels to test the extremal hypothesis of (White, Bettess and Paris [16]), and to investigate the cause of the discrepancy between the predicted and measured values. The programme included 30 straight bankfull-flow tests with flow ranges from 2 to 6 lit/sec and slopes around 0.0017m/m and 0.00214m/m. They were predicted using the White, Bettess & Paris theory [16] (WBP method). The initial bankfull flow sections were trapezoidal with side slope z = 1 (1 vertical: z horizontal). Sediment was bed load only and recirculated through all tests. A bankfull flow channel was left to develop freely until the rate of widening was less than 2% per hour, which suggests an equilibrium condition (see Babaeyan-Koopei and Valentine [3]). These experiments were carried out in a flume 22m long, 2.5m wide and 0.6m deep filled with uniform sand of 1mm median size.

EXPERIMENTAL RESULTS

1. Bankfull Flow Channels

It was found that the resulting bankfull-flow channels remained straight, with plane beds. When compared with the WBP method, the discrepancies between the predicted and measured values (predicted/measured) for width, depth and sediment concentration are 0.50, 1.96 and 0.38 respectively while the area is well approximated. Similar discrepancy ratios have been reported by (Ayyoubzada [2], Babaeyan-Koopei & Valentine [3, 4, 5, 6], Benson et al. [7], Shakir [12]). In this comparison 50 laboratory runs were used with flows ranging from 2 to 52 lit/sec and slopes ranging from 0.001 to 0.003. These data derived from the present laboratory study, with 20 tests taken from the work of others (Ayyoubzada [2], Babaeyan-Koopei & Valentine [6], Benson et al. [7]). Some of these discrepancies are attributed to the formulation followed by the WBP method, which is discussed below.

2. The WBP Method and the Regime Channel Design Tables

The White et al. [16] analytical method describes the channel system using six variables: the average velocity, V, average depth, d, channel slope, S, discharge, Q, sediment concentration, X, and channel width, B. There are four formulae to relate these variables: the resistance formula (White et al. [14]), the sediment transport formula (Ackers and White [1]), the flow continuity equation ($Q = AV$) and the extremal assumes that the sediment rate transported by the channel is a maximum, or equivalently, the stream power is a minimum. As noted, this extremal hypothesis forms the fourth necessary relationship to determine channel width and to predict the regime or equilibrium channel condition.

Flow and slope are assumed to be imposed and the corresponding values of V, d, X, and B can be determined. When the condition of maximum sediment transport is imposed there is only one solution; otherwise there are a family of solutions each with different values of B, X, U and d. Implicit in this approach is the assumption that the channel is wide, the flow is steady and uniform and the bed and bank material is non-cohesive.

White et al. [16] reported that their equations were originally based on flume experiments in which the channel shape was rectangular. As a result, they transformed the rectangular section into an equivalent trapezoidal section. The depth, d, in the resistance equation was replaced with the hydraulic radius, R, and the shear velocity was determined using the relationship, $V_* = \sqrt{gRS}$. The values of width and depth of the predicted rectangular section were then adjusted to give values corresponding to a trapezoidal section of the same cross-sectional area. For determining the side slope, z, of the trapezoidal channel they used Smith's equation [13],

$$z = \begin{cases} 0.5 & if \quad Q < 1\, m^3/\sec \\ 0.5Q^{0.25} & if \quad Q \geq 1\, m^3/\sec \end{cases} \tag{1}$$

Based on this, White, Paris and Bettess [15] produced Design Tables for stable alluvial channels, referred here to as (WPBDT).

3. Computed Geometric Shape

The original shape computed by the WBP method is the rectangular section (shown in Fig. 1(a) by a broken line) with width Br and depth d. Because the channels in nature are better approximated as trapezoidal in shape, it is required to transfer this rectangular shape into an equal-area (equivalent) trapezoidal section with the same depth d and with a top width B, a bottom width b and a side slope z. This can be obtained by assuming,

$$B = B_r + zd \tag{2}$$

$$B_r = b + zd \tag{3}$$

The wetted perimeters of the rectangular and the trapezoidal section can be calculated from Fig. 1(a) as:

- For the rectangular section

$$P_{rect.} = B_r + 2d \tag{4}$$

By replacing B_r using Eq. 3 it can be written as

$$P_{rect.} = b + 2d(1 + z/2) \tag{4a}$$

or, by using Eqs. 2 & 3 it can be written in terms of the top width as

$$P_{rect.} = B + 2d(1 - z/2) \tag{4b}$$

- For the trapezoidal section

$$P_{trap.} = b + 2\sqrt{d^2 + (zd)^2} \tag{5}$$

or it can be written as:

$$P_{trap.} = b + 2d\sqrt{1 + z^2} \tag{5a}$$

By using Eqs. 2 & 3 it can be written in terms of the top width as

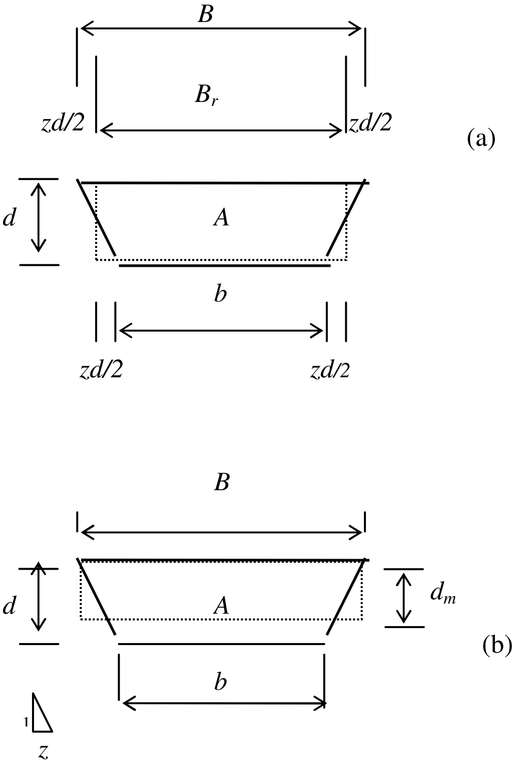

Fig. 1: Transforming a rectangular section into an equal-area trapezoidal section

$$P_{trap.} = B + 2d(\sqrt{1+z^2} - z) \tag{5b}$$

Using Eqs. 4 and 5, the wetted perimeter ratios in terms of bottom and top width will be:

$$\frac{P_{trap.}}{P_{rect.}} = \frac{b + 2d\sqrt{1+z^2}}{b + 2d(1 + z/2)} = \frac{\frac{b}{d} + 2\sqrt{1+z^2}}{\frac{b}{d} + 2(1 + z/2)} \tag{6a}$$

$$\frac{P_{trap.}}{P_{rect.}} = \frac{B + 2d(\sqrt{1+z^2} - z)}{B + 2d(1 - z/2)} = \frac{\frac{B}{d} + 2(\sqrt{1+z^2} - z)}{\frac{B}{d} + 2(1 - z/2)} \tag{6b}$$

3.1. Wide Trapezoidal Channels

For the equivalent trapezoidal channel, assume that when Ptrap/Prect ≤ 1.15, in Eqs. 6, the channel is still wide and hence Rtrap ≈ Rrect. When B/d = 10.0, the ratio P_{trap}/P_{rect} ≤ 1.15 is only achieved if z ≤ 3.0 and the aspect ratio related to the bottom width b/d ≥ 4.0. Assuming a wide channel, B/d ≥ 10.0, White et al. [15] used Eq. 1 to convert the rectangular section in Fig. 1(a) into an equivalent trapezoidal section that has the same hydraulic parameters of the rectangular section (V, B, d, f and X) and published the WPBDT. Nevertheless, this trapezoidal section (in the WPBDT) is tabulated in terms of another equal-area rectangular section, as shown in Fig. 1(b), with the same top width of the trapezoidal section, B, but in terms of the hydraulic mean depth dm where,

$$B_r d = B d_m = A \tag{7a}$$

or

$$B_r = A/d \tag{7b}$$

Substituting Eq. 7b into Eq. 2 the following is obtained:

$$\left.\begin{aligned} &B = A/d + zd \\ &zd^2 - Bd + A = 0 \\ &d = \frac{B \mp \sqrt{B^2 - 4zA}}{2z} \end{aligned}\right\} \tag{8}$$

To use the WPBDT for design of an alluvial trapezoidal channel the depth d has to be calculated first using Eq. 8 with the desired side slope z, but z must be ≤ 0.25 B/dm to allow a solution (see Ayyuobzadeh [2], Brown [8]). The bottom width of the trapezoidal channel then can be calculated by the equation,

$$b = B - 2zd \tag{9}$$

which is a combination of Eqs. 2 and 3.

In Eq. 9, as the top width, B, is kept constant any increase in the z value will cause a decrease in bottom width, b, and hence the aspect ratio b/d leading to a narrower channel until reaching a triangular shape when B/d = 2z or B/d m = 4z. Therefore, to sustain a wide channel, any increase in the z value must be associated with an increase in the top and bottom widths.

3.2. Narrow Trapezoidal Channels

For narrow channels, the wetted perimeters of the trapezoidal and the equivalent rectangular sections are not equal ($P_{trap} \neq P_{rect.}$). Therefore, if the section in the WBP method is considered to be trapezoidal the result will not be identical to that of the equivalent rectangular section, and hence a correction is required to account for the difference in the wetted perimeters.

In order to obtain a general picture of the difference between the wetted perimeters in the equivalent rectangular and trapezoidal sections, equations (Eqs. 4a and 5a) are plotted in Fig. 2 assuming constant width and unit depth.

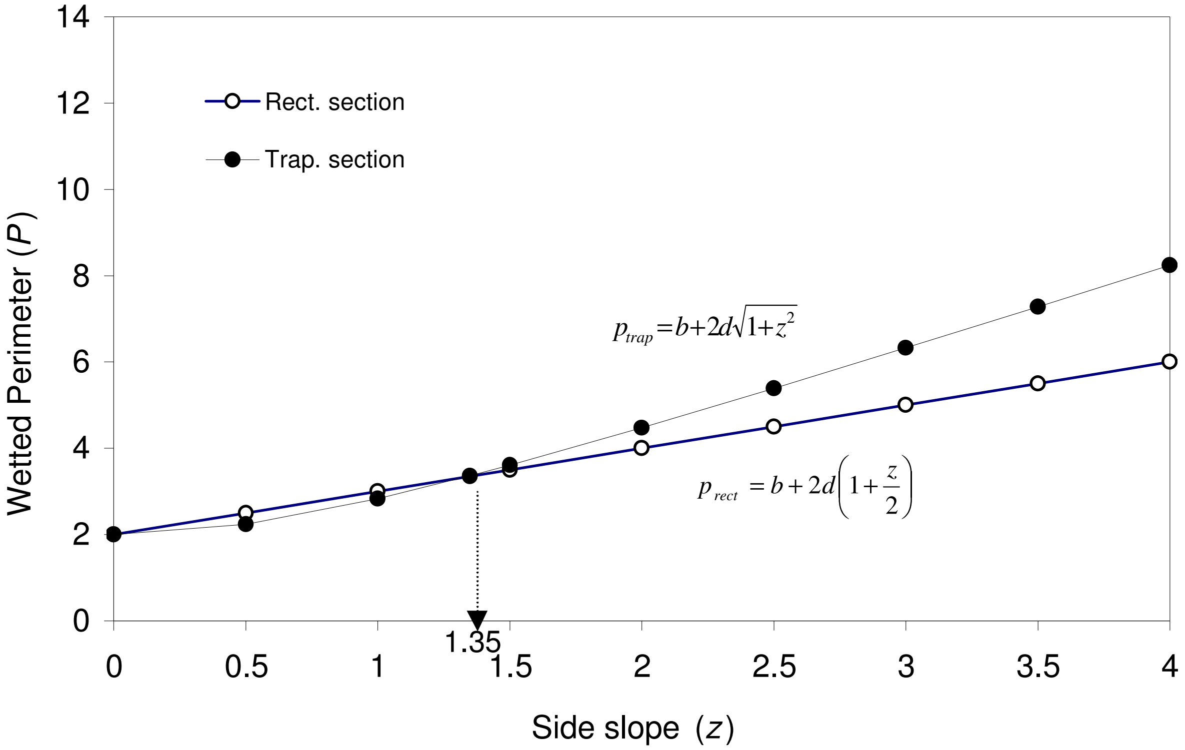

Fig. 2: Comparison between the wetted perimeters of equal-area trapezoidal and rectangular sections

It can be seen that at steeper side slopes, 0.0 < z < 1.35, the wetted perimeter of the rectangular section is slightly larger than that of the trapezoidal section with a maximum value at z = 0.5. This perimeter then decreases up to a certain side slope value, 1.35, where it becomes approximately equal to that of the trapezoidal section. Above this value, they deviate again where the wetted perimeter of the trapezoidal section now becomes greater than that of the rectangular section. As the side slope becomes flatter, the difference between the two perimeters increases further. The hydraulic radius changes in the opposite direction.

It can be said that when $0.0 < z \leq 1.35$, $P_{trap.} < P_{rect.}$, (or $R_{trap.} > R_{rect.}$). Thus, it can be concluded that, for narrow channels the WBP method predicts identical hydraulic parameters for the equivalent trapezoidal and rectangular sections only when the side slope is approximately equal to 1.35.

INFLUENCE OF HYDRAULIC RADIUS ON THE PREDICTED PARAMETERS

A computer programme was written in Fortran, which adopts the WBP theory (maximisation of sediment transport). In this programme the input parameters are the sediment size, D_{35}, slope, S, initial bottom width, bi, initial depth, di, side slope, z, and the flow rate, *Q*. The outputs are the stable bottom width, *b*, top width, *B*, depth, *d*, mean depth, d_m, friction factor, *f*, velocity, *V*, and the maximum sediment concentration, X_{max}. In order to see how different formulations of the equations can produce different results, this programme was written in two versions. In the first version, the same formulation used by White *et al.* [15] to produce the WPBDT, was followed and referred here to as the White, Bettess and Paris Original method (WBPO). In other words, the section is first considered to be rectangular, where the hydraulic radius of the rectangular section, R_{rect} is used for calculating the shear velocity V_* and the mobility number $F_{gr,}$ then converted to an equivalent trapezoidal section which gives results similar to those in the WPBDT. The second version was made by considering the section to be basically trapezoidal and referred here to as the White, Bettess and Paris Trapezoidal method (WBPT). In other words, the hydraulic radius of the trapezoidal section, R_{trap}, is used in calculating the shear velocity, V_*, the mobility number F_{gr} and hence the sediment concentration. This version was made in order to understand the difference between the result of the completely trapezoidal section (WBPT) and the transformed one (WBPO).

For sand beds and flows less than 0.5 m^3/sec, it was found that in laboratory channels z is always larger than the Smith's equation [13] constant value (0.5), which is adopted by the WBP method. In laboratory channels it was found that z ranges from 2 to 3.0. Accordingly, optimisation curves for a narrow channel (Fig. 3) with sediment size D_{35} = 1.0mm are plotted with various side slopes from z = 0.0 to z = 2.0 using the two WBP programs (WBPO and WBPT). This was carried out in order to observe the effect of the side slope changes of narrow and wide channels with a constant flow, on the prediction of the hydraulic parameters, width and sediment concentration.

1. Channel Width

Usually, most of the investigators (e.g. Ayyoubzada [2], Babaeyan-Koopei & Valentine [3, 4, 5, 6], Benson et al. [7], Shakir [12]) have compared their results in laboratory channels, which have side slope z > 2.0, with those in the WPBDT which are tabulated with side slope of 0.5. This gives misleading results especially for the width, where it was found that the measured width was 1.7 to 2.0 times that predicted. Thus, for a fair comparison, width had to be compared using the actual side slope z. This is not possible, because in the WPBDT the side slope z ≥ 2.0 is only associated with flow rate Q ≥ 256 m^3/sec, which is not the case in laboratory channels. In order to see the effect of ignoring the side slopes in any comparison refer to Fig. 3. For example, in Fig. 3a at side slope z = 2.0 the top width B = 0.352m is about 30% greater than that at side slope z = 0.5 (i.e. B = 0.274m). Thus, as the channel in Fig. 3

is narrow (B/d = 5.6), ignoring the actual side slope in the comparison leads to an error of around 30% in width prediction.

2. Sediment Concentration

According to the original WBP formulation (WBPO), Fig. 3a shows that the predicted sediment concentration is constant (80ppm), whatever the side slope. On other hand, when the channel is originally considered to be trapezoidal (WBPT), Fig. 3b shows that the sediment concentration is not constant, but changes with the change of side slope. For example, the sediment concentration of the rectangular section (at $z = 0.0$) is 80.0 ppm, but when the side slope changes to $z = 0.5$ it increases to 91.4 ppm, an increase of 14%. When the value of z increases further the sediment concentration begins to decrease until reaching the same value as the rectangular section at side slope $z = 1.35$.

This illustrates that, at side slope $z = 1.35$, the equal-area rectangular and trapezoidal sections become identical in their characteristics and hence have the same hydraulic parameters. When the z value increases beyond 1.35 the sediment concentration decreases again and the reduction at $z = 2.0$ is about 20% of its original value (at $z = 0.0$). Thus, when the side slope $0.0 < z \leq 1.35$, narrow trapezoidal channels transport more sediment than the equivalent rectangular channels do with a peak value at $z = 0.5$.

Thus, in this range of side slope the WBP method underestimates the sediment concentration. Above side slope $z > 1.35$ narrow trapezoidal channels transport less sediment than the equivalent rectangular channels.

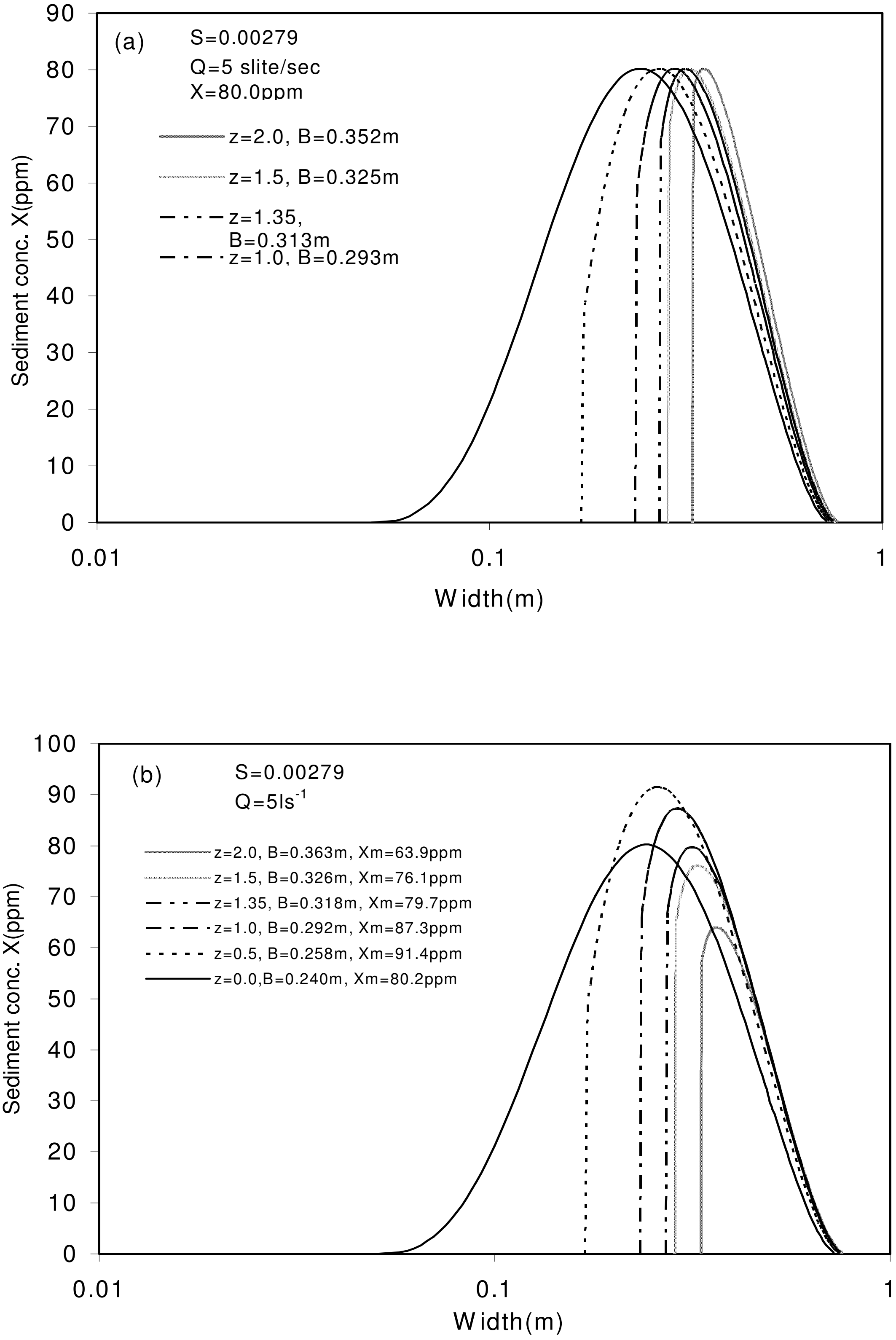

Fig. 3: Optimisation curves for a narrow channel according to the WBP method:
(a) The WBP original method simulations [WBPO].
(b) The WBP trapezoidal-section method simulations [WBPT].

Based on this, narrow channels with side slope $0.0 < z \leq 1.35$ should be treated completely as a trapezoidal section and above that slope the methodology followed by White et al. [15] is satisfactory. The other error in underestimating the sediment by the WBP method is due to the higher value for the threshold of motion obtained by the Ackers-White incipient motion equation [1]. In the present experiments it was found that sediment starts to move at lower values than those predicted by the Ackers-White incipient motion equation and hence this equation needs to be reviewed. When this equation is reviewed using 20 datasets from the present study, which were not used in the final verification, it was found that the coefficient, 0.14, should be replaced by 0.125.

CONCLUSIONS

(1) Underestimation of width and hence the overestimation of depth by the WBP method is mostly attributed to using Smith's formula [13] which does not accurately predict the side slope. Ignoring the actual side slope in the comparison with the WBP method leads to an error of about 30% in width prediction.

(2) There are two reasons why the sediment concentration is underestimated in the WBP method. These are the higher value of the predicted threshold of motion, and the use of the hydraulic radius of the rectangular section in the formulation. For narrow channels this leads to underestimation of sediment transport at side slope $0.0 < z \leq 1.35$, because $R_{rect} < R_{trap}$. In order to increase the mobility number and hence the sediment transport, the hydraulic radius of the trapezoidal section should be used in the formulation at side slope $0.0 < z \leq 1.35$ and outside that range, i.e. at $z \geq 1.35$ the hydraulic radius of the rectangular section should be used instead. For wide channels, no matter which hydraulic radius is used in the formulation, there is clearly little effect.

(3) The Ackers-White incipient motion equation overestimates the threshold of motion and could be revised for wider ranges of data. An attempt was made to evaluate the constant term of the Ackers-White incipient motion equation using the present limited dataset and it was found that the constant 0.14 should be replaced by 0.125.

(4) The WBP method was modified according to (2) and (3) then compared using the remaining 30 laboratory tests. It was found, as a result, that the discrepancy between the predicted and measured values for width, depth and sediment concentration are 0.76, 1.35 and 0.78 respectively instead of 0.50, 1.96 and 0.38 using the WBP original method. Generally, by using the present modification, the differences between the predicted and measured values of width, depth and sediment concentration are much reduced and are within ± 30%.

Finally, it can be concluded that the White *et al.* [16] analytical method is still an excellent tool for practicing engineers in the design of regime channels and is improved at small scales and in narrow channels by the proposed modification. Further work is in progress to test the modification with field data. Both uniform and graded bed materials will be considered.

REFERENCES

[1] P. Ackers, and W.R.White. Sediment transport: new approach and analysis. *ASCE, Journal of the Hydraulics Div.,* Vol. 99, HY11, pp 2041-2060, 1973.

[2] S.A. Ayyuobzadeh. Hydraulic aspects of straight compound channel flow and bed sediment transport. *Ph.D. Thesis, The University of Birmingham,* UK, 1997.

[3] K. Babaeyan-Koopaei and E. M. Valentine. Experimental assessment of rational regime theory. *The 26th Congress of IAHR, Hydra-2000,* Vol. 1, pp 360-365, Thomas Telford London. 1995a

[4] K. Babaeyan-Koopaei and E. M. Valentine. A rational regime theory: different combinations of formulae. *The Second Int. Conf. on Hydro-Science and Eng.*, Vol. 2, pp 2037-2044, Beijing, China, 1995b

[5] K. Babaeyan-Koopaei and E. M. Valentine. Assessment of an updated rational regime theory. *The 27th Congress of IAHR, Vol. 2, pp 985-990,* San Francisco, California-USA. 1997.

[6] K. Babaeyan-Koopaei. A study of straight stable channels and their interactions with bridge structures. *Ph.D. Thesis, University of Newcastle Upon Tyne,* UK. 1996.

[7] I. Benson, E.M Valentine, C. Nalluri and J.C. Bathurst. Flood Channel Facility: Experiments in two-stage channels mobile boundary channels. *The 27th Congress of IAHR,* Vol. 2, pp 949-954, San Francisco, California-USA, 1997.

[8] F.A Brown. Sediment transport in river channels at high stage. *Ph.D. Thesis, The University of Birmingham,* UK, 1997.

[9] H.H. Chang. Minimum stream power and river channels patterns. *Journal of Hydrology,* 41, pp 303-327, 1979.

[10] P. Diplas. Hydraulic geometry of threshold channels, *Journal of Hydraulic Engineering, ASCE,* Vol. 18, No. 4, 1992.

[11] G. Lacey. Stable channels in alluvium. *Minutes of Proc. ICE, 229, pp 259-384,* 1929-30.

[12] A.S. Shakir. An experimental investigation of channel plan forms. *Ph.D. Thesis, University of Newcastle Upon Tyne*, UK, 1992.

[13] K.V.H. Smith. Comparison of prediction techniques with records of observations on the Lower Chenab canal system. *Report No. CE/5/74, University of Southampton,* UK, 1974.

[14] W.R. White, E. Paris, and R. Bettess. The frictional characteristics of alluvial streams: a new approach. *Proc. ICE*, Part 2, 69, pp 737-750, 1980.

[15] W.R. White, E. Paris, and R. Bettess. Tables for the design of stable alluvial channels. *Hydraulics Research,* Wallingford, Report IT 208, 1981.

[16] White W.R., Bettess R. and E. Paris. Analytical approach to river regime. *ASCE, Journal of the Hydraulics Div.,* 108, HY10, pp 1179-1193, 1982.

[17] M.S. Yalin. *River Mechanics*. Pergamon Press Ltd, 1st Edition, Oxford, England, pp 8-14, 1992.

[18] M.S. Yalin and Da Silva A.M. Ferreira. Regime channels in cohesionless alluvium. *Journal of Hydraulic Research*, Vol.37, No.6. 1999.