DISCHARGE MEASUREMENT IN TRAPEZOIDAL LINED CANALS UTILIZING HORIZONTAL AND VERTICAL TRANSITIONS

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ABSTRACT

Most of previous works for discharge measurement structures were carried out in canals with rectangular cross sections. However, in irrigation systems most common irrigation canals, especially lined canals, have trapezoidal cross sections. From practical view point, it is preferable to construct measuring structures without changing the side slopes of the original canals. Usually, a horizontal contraction is used as a permanent measuring device and a vertical hump as temporarily measuring device. In contrary to rectangular contraction, direct analytic solution of head-discharge relation is not possible as the governing equations are implicit. The solution requires tedious methods of trial and error. Tabular and graphical methods are also available for solution which are subject to errors of double interpolation and errors of judgment in reading the graphs. Reported herein are explicit equations for head-discharge relation for both horizontal contraction and vertical hump in trapezoidal canals without changing the side slope of canal. Both horizontal and vertical transitions in the trapezoidal canals are analyzed by the one dimensional momentum equation. An optimization tool is used to compute the critical depth and critical width simultaneously for horizontal contraction, and the critical height of the hump and critical depth for vertical transition. Where the objective function is to adjust the specific energy at the contraction equal to the specific energy at the approach channel according the constraint of the Froude number at contraction equal to 1.0. An explicit approximate equation was developed for the critical depth and other equation was developed for the critical width in the case of horizontal contraction. Other equations were developed for computing the critical height of hump and the critical depth over it. A general depth-discharge relationship was obtained for both types of transitions. Predicted discharges using this general relationship were compared with experimental data and gave a good agreement.

Key words: *trapezoidal cross-section, contraction, vertical hump, discharge measurement.*

INTRODUCTION

We are at the point where our water supply is being critically examined to determine quantity, use, and waste. Planes must therefore be formulated for extending the use of present supplies. One way to increase the quantity of water is to find new water sources. This is not always possible and is usually cost prohibitive. Another method is to conserve and equitably distribute the water presently available. The first step in this process is to establish how much water or flow is available for use; thus, measuring water in open channels is critical towards water conservation.

For discharge measurements in open channels, for example in irrigation canals, sewage treatment plants or industrial water supply systems venturi flumes and weirs are often used. These devices are described in the literature, e.g. Bos [1], Barczewski and Juraschek [2]. The present knowledge on rectangular venturi flumes is large. As regards to trapezoidal venturi flume, however, there is practically a little work except of Robinson and Chamberlain [3]. Based upon the study of Robinson and Chamberlain [3], the trapezoidal flumes have many characteristics which are listed as follows: (1) extreme approach conditions seem to have a minor effect upon head-discharge relationships; (2) material deposited in the approach did not change the head-discharge relationships noticeably; (3) a large range of flows can be measured through the structure with a comparatively small change in head; (4) the flumes will operate under greater submergence than rectangular shaped ones without corrections being necessary to determine the exact discharge; (5) the trapezoidal shape fits the common canal section more closely than a rectangular one; (6) construction details such as transitions and form work are simplified. Hansen et al. [4] showed that in spite of the relationship between head and discharge is not as easily expressed in the form of an equation as is the rectangular shaper flume, it has the advantage that a small change in head results in a comparatively large change in discharge and thus the sensitivity of the flume to changes in discharge is less than that for the rectangular one. Hager [5,6]; and Samani& Magallanez [7] showed that a cylinder or a circular cone installed axially in a prismatic channel can be used to measure the discharge.

The purpose of the present research is to develop the necessary explicit equations for permanent and temporarily computation of the discharge in trapezoidal channel by using horizontal and vertical contractions respectively without change of the shape of the channel.

GOVERNING EQUATIONS

The flow movement through the contracted flume as shown in Fig. 1 can be defined using the conventional energy equation and the Froude number (F_e) relationships. Assuming a uniform velocity distribution, the specific energy equation upstream of the critical flow section can be written as;

$$E_{s1} = y_1 + \frac{0.1Q^2}{2gA^2}$$

in which E_{s1} is the specific energy upstream of the critical flow section; y_1 is the depth of the water upstream; Q is the flow rate; and A is cross-sectional area of flow upstream of the critical flow section.

Assuming a level contracted flume, and negligible energy loss between upstream and critical flow section, the upstream energy will be equal to the energy at the critical section, and can be described as

$$E_{s1} = E_c = y_c + \frac{Q^2}{2gA_c^2} \qquad (2)$$

in which E_c is the energy at the critical flow section; y_c is the distance from the surface of the water at the critical point to the flume floor; Q is the flow rate; and A_c is the critical flow cross section.

The water will reach the critical flow at the smallest cross section for the canal. Since critical flow occurs with Froude number equal to 1, the critical flow equation can be described as

$$\frac{Q^2}{gA_c^3}\left(\frac{\partial A_c}{\partial y_c}\right) = F_e^{\ 2} = 1 \qquad (3)$$

in which $\partial A_c / \partial y_c$ represents the derivative of critical flow cross section with respect to y_c.

By rearrangement of Eqn. 2, the discharge equation can be written as follow;

$$Q = \left(b_c y_c + y_c^2 / \tan\theta\right)\sqrt{2g(E_{s1} - y_c)} \qquad (4)$$

in which b_c is the bottom width at the critical flow section; y_c is the distance from the surface of the water at the critical point to the flume floor; Q is the flow rate; and θ is the side slope angle of flow cross section in degrees.

At the case of rectangular cross section $y_c^2 / \tan\theta = 0$ and $y_c = 2/3E_{s1}$ which lead to the well known formula;

$$Q = 0.544 b_c \sqrt{g} E_{s1}^{3/2} \qquad (5)$$

By the same way, at the case of vertical transistion shown in Fig. 2, Eqn. 4 can be written as follow;

$$Q = \left(\left[B + 2Z_c / \tan\theta\right] y_c + y_c^2 / \tan\theta\right)\sqrt{2g(E_{s1} - (y_c + Z_c)} \qquad (6)$$

in which B is the bottom width of the channel at the upstream flow section; y_c is the critical depth of water over the hump; Q is the flow rate; and Z_c is the hump height which produces critical flow over it.

In the case of trapezoidal cross section, values of y_c, b_c and Z_c depend on values of B, E_{s1} and θ. In the following sections, explicit equations for computing y_c, b_c and Z_c will be formulated.

DIMENSIONAL ANALYSIS

Normally the discharge Q is determined by measuring the upstream water depth y_1 at a position of one to three times the maximum water depth upstream of the inlet distortion, (Fig. 1,2). A physically pertinent relation between the discharge and the upstream water depth, that means the type of the rating curve, may be found by dimensional analysis. The non-dimensional relationship is also useful for checking the sensitivity of the different parameters which affect the phenomenon, Keller [8].

The functional relationship of the discharge Q in the case of horizontal contraction may be expressed by:

$$Q = f(B, b_c, E_{s1}, y_c, \theta, g, \rho) \qquad (7)$$

and for vertical contraction, Fig. 2:

$$Q = f(B, Z_c, E_{s1}, y_c, \theta, g, \rho) \qquad (8)$$

where B, b_c are the widths of the bottom of the upstream channel and of the throat, y_c is the critical water depth at the throat, E_{s1} is the specific energy at upstream channel, θ is the side slope angle of the channel, g is the acceleration due to gravity, ρ is the density, and Z_c is the critical height of the hump. In practical design, the effect of the velocity of approach may be entirely disregarded and E_{s1} is replaced by the water depth in the approach channel.

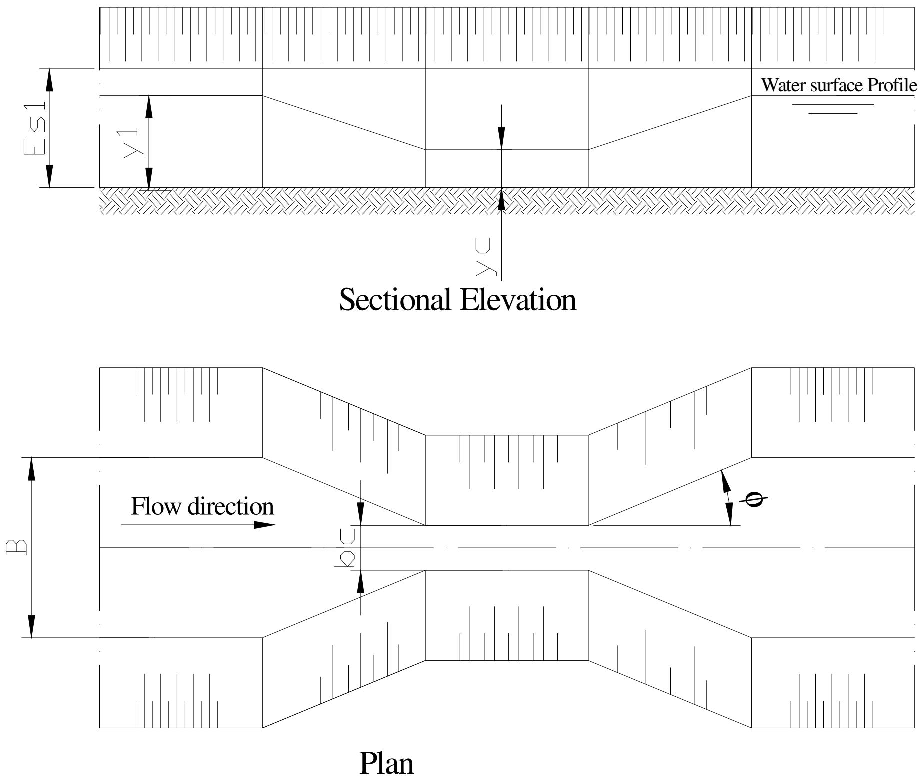

Fig. (1): Definition sketch for the horizontal contraction in trapezoidal channel

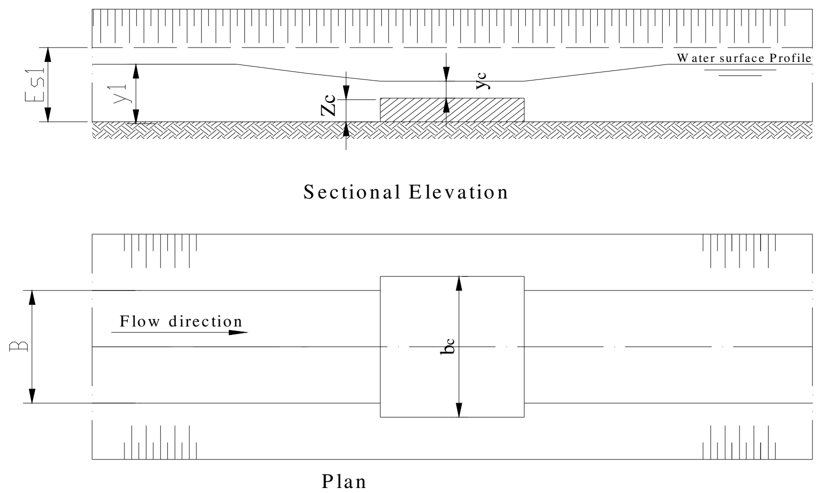

Fig. (2): Definition sketch for the vertical transition in trapezoidal channel

Some transformations lead to the non-dimensional relation for both horizontal and vertical transitions respectively.

$$\frac{Q}{\sqrt{g}.E_{s1}^{2.5}} = f1\left(\frac{B}{E_{s1}}, \frac{b_c}{E_{s1}}, \frac{y_c}{E_{s1}}, \theta\right) \tag{9}$$

and

$$\frac{Q}{\sqrt{g}.E_{s1}^{2.5}} = f1\left(\frac{B}{E_{s1}}, \frac{Z_c}{E_{s1}}, \frac{y_c}{E_{s1}}, \theta\right) \tag{10}$$

In the following section the four groups on the right hand side of Eqns. 9 and 10 will be correlated in two explicit groups. Dimensionally consistent equations have been obtained for critical depth, critical width and height of hump in the following sections. Using these equations it has been possible to obtain explicit discharge equations without need to iteration.

EXPERIMENTAL DATA

The experimental data in this study were carried out by Robison and Chamberlain [3] on a trapezoidal flume with horizontal contraction where diversion and conversion angle ϕ was kept constant and equal to about 10^o. The side slope angle θ was changed three times and the width of the channel was changed two times. The flowing discharge was in the range from 1.4 to 57 l/sec. and the water depth in the approach channel was in the range from 3.5 to 26 cm. Table (1) shows the range of used variables. These data are used as a boundary conditions in the approach channel while the critical width and depth of contraction are computed using equation 2. The same thing is carried out for vertical hump where the critical height of hump and critical flow depth are computed. An optimization tool is used to compute the critical depth and critical width simultaneously for horizontal transition, and the critical height of the hump and critical depth for vertical contraction. Where the objective function is to adjust the specific energy at the contraction equal to the specific energy at the approach channel according the constraint that Froude number at contraction equal to 1.0.

Table (1): Range of used experimental data.

Flume No.	B (cm)	θ^o	Q (l/s)	y_1 (cm)
1	12.4	60	1.4 ~35	4.9 ~24
2	12.4	45	1.4 ~42	4.5 ~21
3	12.4	30	1.4 ~57	3.9 ~20
4	20.3	60	1.4 ~57	3.5 ~26

RESULTS AND DISCUSSIONS

This analysis is based on modular flow, where the contraction is strong enough, the major part of the total upstream energy head E_{s1} will be converted into kinetic energy to obtain critical flow at the control section. Such a section with critical flow is required to make the upstream specific energy head E_{s1} independent of downstream conditions. This flow type is normally referred to as modular flow or free flow, Boiten [9].

1. Horizontal Transition (Contraction)

The horizontal contraction is useful in sewage and irrigation techniques, since un-dissolved matter and sediments are less deposited in the upstream reach, and backwater effects remain small. Where local reduction of channel's cross section, creates a drop in the water level over the contraction. Provided this reduction is strong enough, the major part of the total upstream energy head will be converted into kinetic energy to obtain critical flow at the control section. Such a section with critical flow is required to make the upstream head independent of downstream conditions.

Determination of The Critical Depth

From the dimensional analysis (Eqn. 9), the parameter y_c / E_{s1} is function of the upstream width to upstream energy head ratio (B / E_{s1}) and the side slope angle of the cross section, θ. In order to determine the correlation between these parameters, the results are presented in Fig. 3 in which y_c / E_{s1} is plotted against B / E_{s1} together for the different values of the side slope angles θ. It is noticeable from this figure, that values of y_c / E_{s1} ratio are in the range from 0.69 to 0.78 depending on values of the side slope angle and this differs from the value of 0.66 in the case of rectangular contraction. Also, it is readily seen that the data align logarithmic curves which satisfy the following equation:

$$y_c / E_{s1} = K_1 \ln(B / E_{s1}) + K_2 \qquad (R^2 = 0.95) \qquad (11)$$

in which K_1 and K_2 are coefficients. It was found that K_1 and K_2 due independent of side slope angle θ and take mean values of -.0253 and 0.763 respectively. Then, Eqn. 11 becomes;

$$y_c / E_{s1} = -0.0253 \ln(B / E_{s1}) + 0.763 \qquad (12)$$

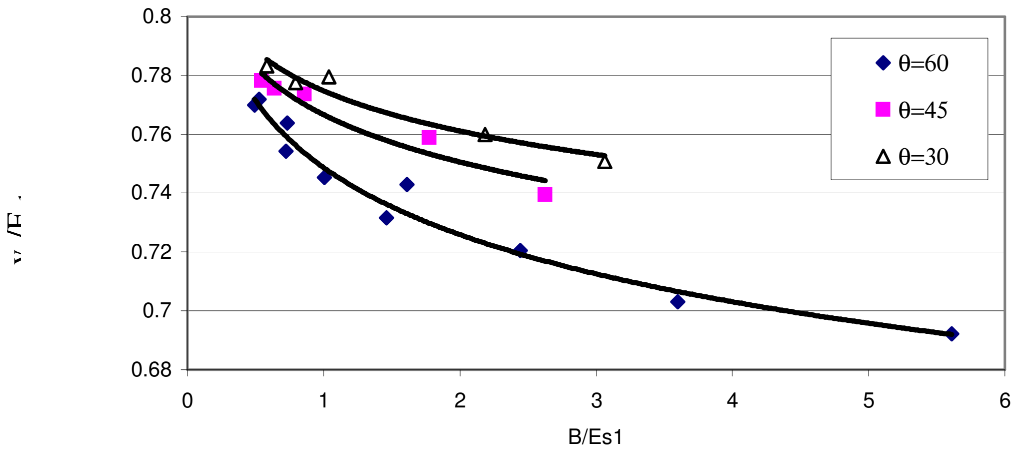

Fig. (3): Values of y_c / E_{s1} versus B / E_{s1} for different values of θ.

Determination of The Critical Width

Figure 4 shows the variation of b_c / E_{s1} with B / E_{s1} for different values of side slope angle θ. Apparently, for all values of θ, the data are scattered around a curve shown in Fig. 5 which can be represented by the following equation:

$$b_c / E_{s1} = 0.042(B / E_{s1})^2 + 0.27(B / E_{s1}) + 0.106 \qquad (R^2 = 0.98) \qquad (13)$$

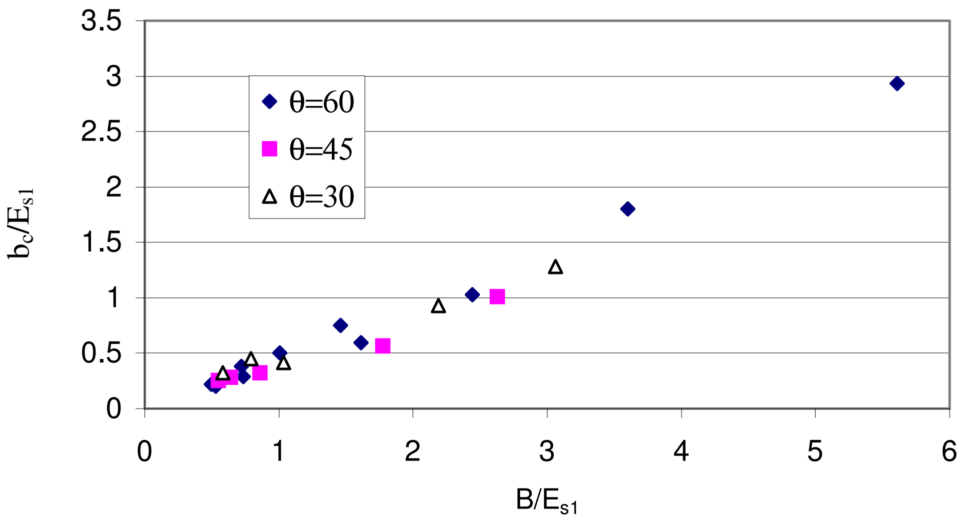

Fig. (4): Values of b_c / E_{s1} versus B / E_{s1} for different values of θ

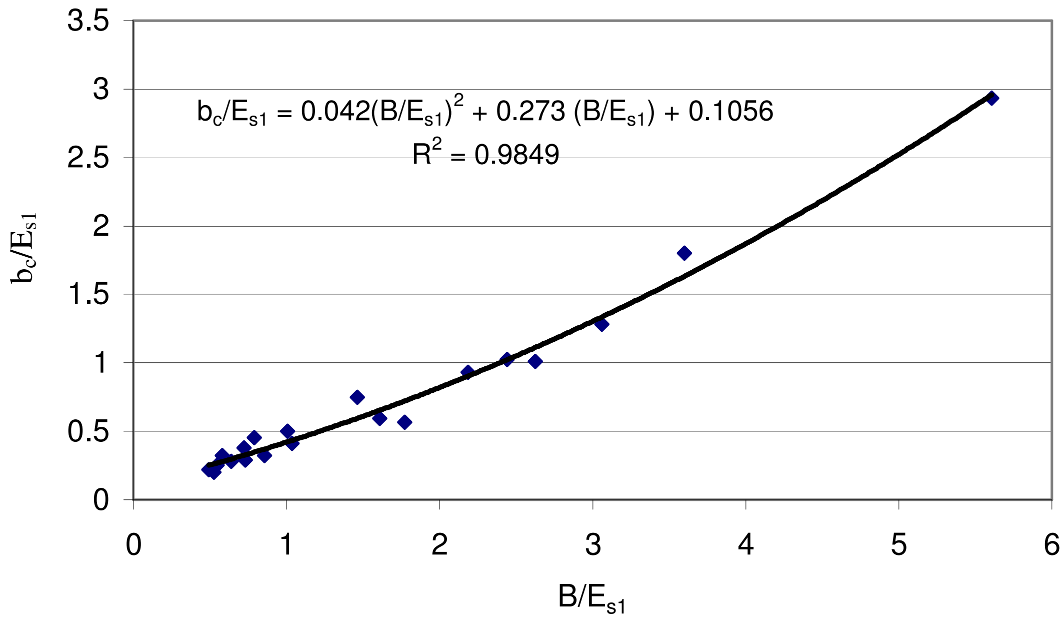

Fig. (5): Correlation equation of b_c / E_{s1}.

Discharge Computations

The discharge can be estimated using the developed equations by knowing the channel bed width and the energy head upstream the contraction which can be approximated by the water depth. The actual measured flow rates were compared with the calculated flow rates using Eqns. 4, 12 and 13. The result of the comparison is given in Fig. 6, which shows that the developed equations can predict the measured flow rate with an error less than 5% .

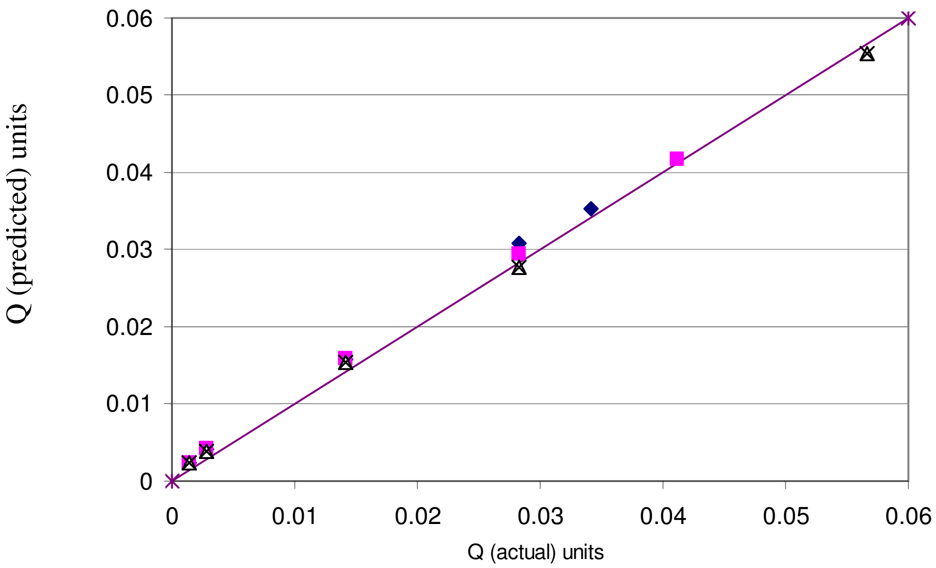

Fig. (6): Predicted versus actual discharges

2. Vertical Transition (Hump)

If discharge have to be recorded at different points in a channel system (such as irrigation or sewage canalizations) only over a limited period, a mobile apparatus would be convenient. Obviously, such an instrument should not change the original cross section and be simply adjustable in the channel. Mobile hump installation often may become significant as shown in Fig. 2. When the hump creates critical condition over it, it can be used for discharge measurements. The vertical transition is more complicated than the horizontal contraction where the width increases with height of the hump and this un-usual case.

Determination of The Critical Height of Hump

In Fig. 7, values of Z_c / E_{s1} are plotted versus B / E_{s1} for each side slope angle θ. The data for each value of θ clustered around a curve which may be expressed in the form;

$$Z_c / E_{s1} = C_1 (B / E_{s1})^2 + C_2 (B / E_{s1}) + C_3 \qquad (R^2 = 0.96) \qquad (14)$$

in which C_1 , C_2 and C_3 are coefficients. It was found that C_1 , C_2 and C_3 to be independent of side slope angle θ and take mean values of -.057, 0.237 and 0.167 respectively. Then, Eqn. 14 becomes;

$$Z_c / E_{s1} = -0.057 (B / E_{s1})^2 + 0.237 (B / E_{s1}) + 0.167 \qquad (15)$$

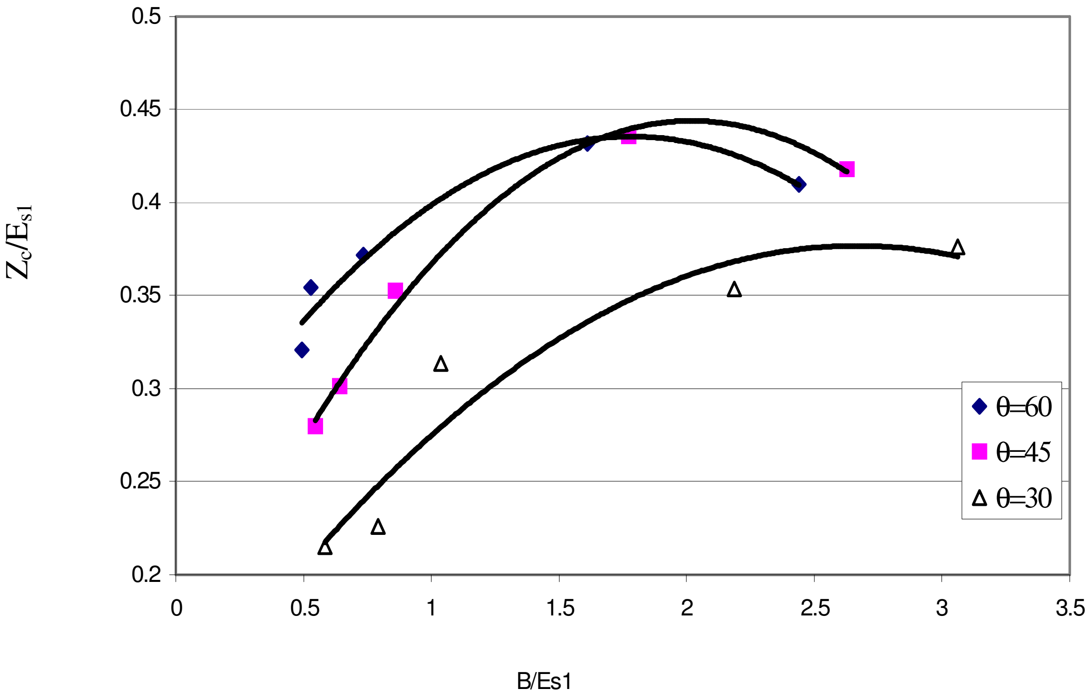

Fig. (7): Values of Z_c / E_{s1} versus B / E_{s1} for different values of θ.

Determination of The Critical Depth

Figure 8 shows the relation between the relative critical depth y_c / E_{s1} and B / E_{s1}. The data for each value of θ are grouped around a curve that may be expressed in relation of the form;

$$y_c / E_{s1} = M_1 \ln(B / E_{s1}) + M_2 \qquad (R^2 = 0.90) \tag{16}$$

in which M_1 and M_2 are coefficients. It was found that M_1 and M_2 to be independent of side slope angle θ and take mean values of -.0748 and 0.474 respectively. Then, Eqn. 16 becomes;

$$y_c / E_{s1} = -0.0748 \ln(B / E_{s1}) + 0.474 \tag{17}$$

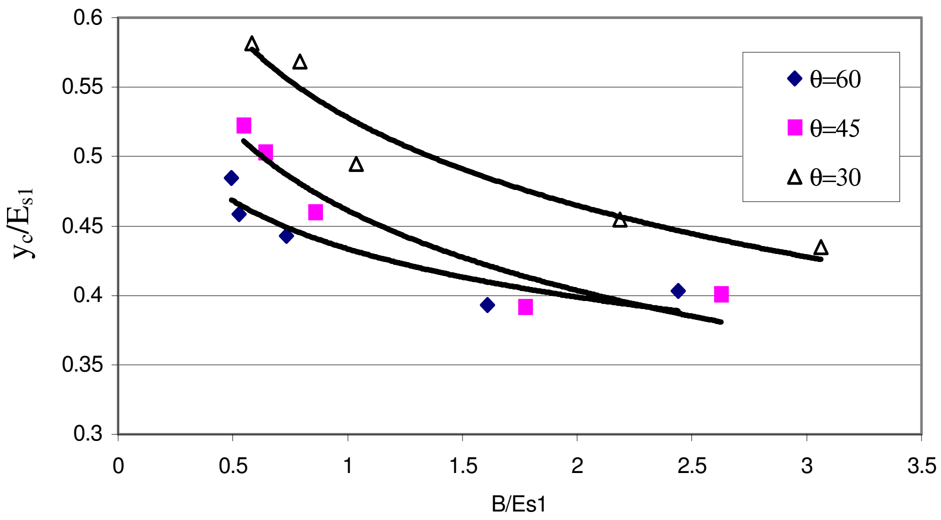

Fig. (8): Values of y_c / E_{s1} versus B / E_{s1} for different values of θ.

Discharge Computations

Now, the discharge can be estimated using the developed equations by knowing the channel bed width and the energy head upstream the contraction which can be approximated by the water depth. The actual measured flow rates were compared with the calculated flow rates using Eqns. 6, 15 and 17. The result of the comparison is given in Fig. 9, which shows that the developed equations can predict the measured flow rate with error less than 5%.

The approach presented herein should be checked with other experimental data, where the effect non-modular flow conditions and conversion and diversion of constriction angle and effect of head loss through the contraction must be taken into account.

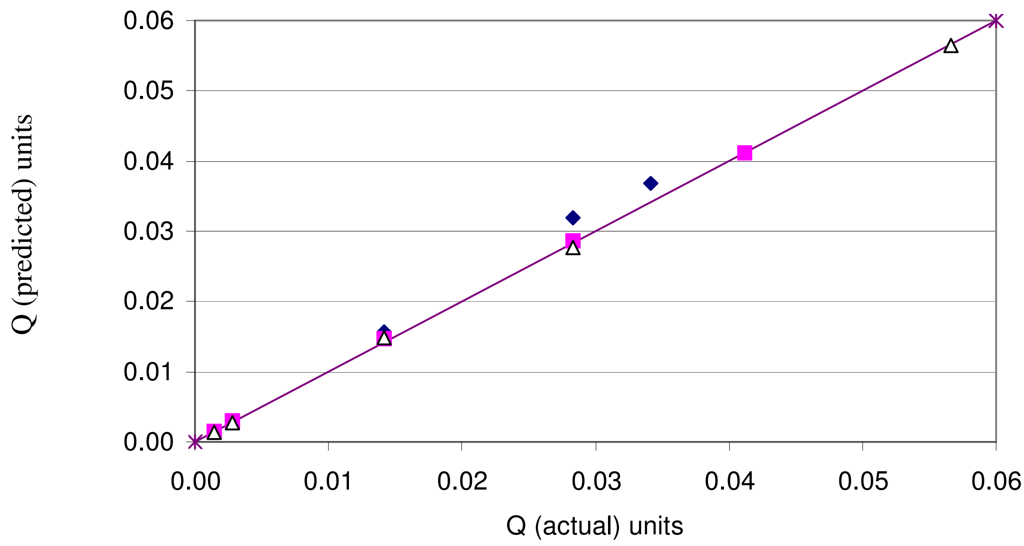

Fig. (9): Predicted versus actual discharges.

CONCLUSIONS

The main conclusions drawn from this study can be summarized as follows:

1. From the present analysis, an explicit relationship has been developed for the discharge computations in trapezoidal canal using either horizontal or vertical transition.
2. In case of horizontal contraction, an equation was developed for the critical width and another one for the critical depth at contraction.
3. For vertical transition, an equation was developed for computing the critical height of the hump and an equation was developed for the critical depth over it.
4. The analysis reveals that the side slope angle of the channel has small effect on critical depth at contraction and also width or height of contraction which can be neglected.
5. Values of critical depth at critical contraction of trapezoidal canal are higher than that at rectangular one which equal to 0.67 with respect to the upstream specific energy.
6. The predicted flow rates based on the proposed approach were compared with the corresponding measured ones. The comparison showed a good agreement.

NOMENCLATURE

A = the cross sectional area at upstream channel

A_c = cross sectional area at critical section

B = width of approach channel

E_c = the energy head at critical section
E_{s1} = the energy head at approach channel
F_e = Froude number
g = gravity of acceleration
Q = the flow rate
V_1 = upstream mean velocity
V_c = critical velocity
y_1 = the water depth at approach channel
y_c = the critical depth at critical section
Z_c = height of hump which produce critical depth
θ = the side slope angle
ρ = the water density

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