EFFECT OF VERTICAL CURVATURE OF FLOW AT WEIR CREST ON DISCHARGE COEFFICIENT

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ABSTRACT

In the present research paper the effect of vertical curvature of flow streamlines due to change in weir crest height on discharge coefficient (Cd) was experimentally studied. The study was carried out on four different types of weirs under different flow conditions (sharp crested weir, broad crested weir, triangular weir, and trapezoidal weir). Also, the study deals with the effect of head angle of the triangle weir on discharge coefficient. This study was carried out through 254 experimental runs. The study showed that the discharge coefficient (Cd) in both sharp and broad crested weirs was affected by the relative crest height (H/Y). By using the regression analysis, two formulas between the discharge coefficient (Cd) and the dimensionless crest height (H/Y) for both sharp and broad crested weirs were presented. The comparative study between the developed formulas and those formulas developed by other researchers showed good agreement in discharge coefficient values. For triangular weir, the effect of head angle (θ) on discharge coefficient (Cd) was more than the effect of relative head (H/Y). Also, the experimental study illustrates that the effect of relative head (H/Y) on discharge coefficient (Cd) in trapezoidal weir could not be considered. This result was consistent with the results of USBR [14].

Keywords: sharp crested weir, broad crested weir, V-notch, discharge coefficient

INTRODUCTION

Since weirs have been used as flow measuring devices, the effect of the relative crest height on discharge coefficient should be cleared. Also, as the head angle forms a main feature of triangular V-notch, its effect on the discharge coefficient (Cd) should be studied. Calculation of the actual discharge over weirs is complicated due to the curvature of the streamlines in both vertical and horizontal directions of the flow. The difference between theoretical and actual performance of the weir could be minimized by using discharge coefficient (Cd), which could be determined from laboratory study. Also, one of the reasons of using coefficient (Cd) is that the flow heads at both upstream and over the crest are not equal due vertical curvature of the streamlines, another reason is due to the assumption that the pressure distribution upstream weir is hydrostatic, which is not true due to experiments as stated by Henderson [5].

The discharge coefficient is defined as the ratio between the true flow rate and the theoretical flow rate. The true flow rate of rectangular sharp crested weir is calculated by the following equation:

$$Q = 2/3\ Cd\ b\ \sqrt{2g}\ H^{3/2} \quad (1)$$

The value of discharge coefficient (Cd) depends on many factors as follows:
$Cd = f(R_e, W_e, H/Y)$

in which

R_e	Reynolds No. = VR/ν;
We	Weber No.= V/(√σ/ ρL);
R	mean hydraulic radius;
Y	crest height over bed; and
H	flow head over weir crest.

Also, the discharge coefficient of weirs may depend on the shape, finishing of surface, degree of aeration, submergence ratio, and contraction, but most of the former studies mentioned that it depends mainly on the curvature of the streamlines, especially the vertical curvature. The vertical curvature of the streamlines depends mainly on the relative crest height (H/Y).

Henderson [5] stated that $Cd = Cc[(1+V_a^2/2gH)^{3/2} - (V_a^2/2gH)^{3/2}]$. He stated that the coefficient Cc and the ratio $V_a^2/2gH$ are dependent on the boundary geometry alone, in particular on the ratio H/Y. He also stated that Rehbock [12] summarized his experimental work in the following formulas

$$Cd = 1.06*(1 + w/H)^{3/2} \qquad \text{for } H/w >= 20$$

$$Cd = \underset{\mathbf{(1)}}{0.611} + \underset{\mathbf{(2)}}{H/305} + \underset{\mathbf{(3)}}{0.08H/Y} \qquad \text{for } H/w <= 5 \quad (2)$$

In the aforementioned relationship, term (**1**) is constant, term (**2**) shows the capillary effect, while term (**3**) contains the effect of the approach velocity [12].

Henderson [5] stated that formula (2) is true only for values of H/Y up to approximately 5, but for 5 < H/Y < 10 the value of Cd begins to diverge from the value given by the formula reaching a value of 1.135 when H/Y = 10. Francis [7] concluded the effect of the horizontal curvature of the streamlines as L_{ef} =L- 0.1nH provided that L > 3H, in which n is the number of side contractions. Empirical relationship of discharge coefficient Cd was developed by Rouse 1946 (source [5]) as:

$$Cd = 0.611 + 0.075\ H/Y \qquad \text{for } H/Y < 5\text{-}10$$

Terry W. Sturm [13] stated that Kindsvater and Carter proposed that Reynolds number and Weber number effects can be included in the discharge equation over sharp crested weir.

$$Q = 2/3\ Cd_e\ b_e\ \sqrt{2g}\ H_e^{3/2} \quad (3)$$

in which:

Cd_e	effective discharge coefficient which depends on H/Y and b/B;
H	head on weir crest;
Y	the crest height;
B	approach channel width;
b	the crest length;
b_e	effective crest length = b + $k_{L;}$
H_e	effective energy head = H + $k_{H.}$;
k_L	a constant depending on b/B; and
k_H	constant value equals to 0.001m [13].

The broad crested weir is a rectangular weir of crest width enough to maintain hydrostatic pressure distribution of the flow across it, in which the flow is apparently to be critical. The broad crested weir is a special case of the sharp crested weir, which could be achieved when 0.08 < H/w < 0.5 [10], in which w is the crest width. Chow [3] stated that for H/w<0.08, the head losses could not be neglected and for H/w > 0.5 the streamlines are not horizontal. In this study the values of H/w ranges between 0.18 to 0.33, while the values of H/Y ranges between 0.32 and 1.47. As stated by Robrett et al. [11], the broad crested weir could be considered as a measurement for critical flow.

$$Q = L.q = L\sqrt{g y_c^3} = (2/3)^{3/2} L\sqrt{g}\, E_c^{3/2} \quad (4)$$

in which $y_c = (q^2/g)^{1/3}$ and $E_c = 1.5\, y_c$

As the broad crested weir has a rectangular flow section, it could be considered a special case of the sharp crested weir, then

$$Q = 2/3\; Cd\; L\sqrt{2g}\, H^{3/2} \quad (5)$$

Equating Eqs. (4) and (5) then

$$Cd = (1/\sqrt{3})(E_c/H)^{3/2} \quad (6)$$

Terry [13] stated that the coefficient of discharge of broad crested weir is a function in H/Y and H/w. In this study the effect of relative head (H/Y) was only considered. Also, Terry [13] stated the following:

0.08<H/w<0.33	broad crested weir
0.33<H/w<1.5	short crested weir
H/w>1.5	sharp crested weir

As stated by Terry [13], in case of broad crested weir the crest width is long enough that parallel flow and critical depth occur at a point along the crest. If the energy equation is applied between the approach flow section and the critical flow section on the crest provided that the energy loss is neglected then

$$H_e = y_c + Q^2/2gA_c^2 = 3/2[(Q/B)^2/g]^{1/3} \quad (7)$$

in which

H_e energy head = H + $v_a^2/2g$.

v_a approach velocity.
A_c critical flow cross-section.

As stated by Terry [13], if the energy loss is compacted in discharge coefficient Cd, the discharge on broad crested weir is computed from rearrange of Eq. 7 as follows:

$$Q = C_v C_d\, (2/3)(2g/3)^{1/2} B H^{3/2} \quad (8)$$

in which
C_v the approach velocity coefficient = $(H_e/H)^{3/2}$

Eq. 8 could be solved for the discharge assuming C_v=1 and head due to approach velocity can be calculated for a second calculation of Q. Terry [13] stated that, Bos [2] studied the approach velocity correction coefficient for a broad crested weir as a function of discharge coefficient (Cd), flow area in control section (A*), and flow cross-section in approach section (A_1). He developed the following relationship:

$$A^*Cd/A_1 = (C_v^{2/3}-1)^{1/2}/0.385C_v \quad (9)$$

The study carried out by the Author [4] on the broad crested weir showed that the discharge coefficient is affected by the shape of crest edges, crest width, and crest surface friction.

Henderson [5] stated that the coefficient of discharge of triangular weir is somewhat less than the coefficient of discharge of rectangular weir depending mainly on head angle (θ). The equation of discharge through triangular weir is obtained from the following equation:

$$Q = 8/15\ Cd\ \tan\theta/2 \sqrt{2g} H^{5/2} \quad (10)$$

The trapezoidal weir is used to compensate the end contraction effect of rectangular weir. The equation of discharge through trapezoidal weir is obtained from the equations of both rectangular and triangular weirs as follows:

$$Q = 2/3\ Cd\ B\sqrt{2g}\ H^{3/2} + 8/15\ Cd\ \sqrt{2g}\ \tan\theta/2\ H^{5/2} \quad (11)$$

EXPERIMENTAL WORK

The experimental tests of the present study was carried out in recirculating plexiglass laboratory flume with smooth timber models of sharp crested weir, broad crested weir, triangular V-Notch and trapezoidal weir. Weir models were manufactured from timber with smooth surface and sealed with non permeable material. The discharge was measured by using flowmeter fitted behind the pump and a calibrated triangular V-notch at flume end. Water depths and weir crest heights was measured by using a point gauge supplied with verniers allowing measurements accuracy of ±0.1 mm. The experimental work was carried out through 254 runs. For each weir type six models with different crest heights were used except the broad crested weir seven different heights models are used as illustrated in photo 1. The crest height was changed from 3.0 cm to 8.0 cm over flume bed except the broad crested weir its crest was changed from 3.0 to 9.0 cm. For each crest height six different values of discharge are used. The head over crest was changed from 2.4 cm to 4.75 cm for rectangular sharp crested weir, 2.65 to 5.0 cm for broad crested weir, 5.3 to 7.8 for triangular V-notch, and 3.1 to 5.1 for trapezoidal weir according to change in flow over weir. For the broad crested weir, the crest was long enough that parallel flow and critical depth occur at some point along the crest (w=15 cm). For the triangular weir four values of head angles were used (43^o, 36^0, 31^0, 27^0). The trapezoidal models were manufactured with side slope 1H:4V to compensate the effect of side contractions in rectangular type [6].

The ratio H/Y was used to define the height of weir crest. Tests were conducted for sharp crested weir with H/Y ranges from 0.375 to 1.47, while for broad crest weir, H/Y ranges from 0.32 to 1.47. For triangular V-notch, H/Y ranged from 0.69 to 2.4, while for trapezoidal weir the values of H/Y ranged from 0.39 to 1.7.

Before experiments were carried out the flume was leveled horizontally and the flowmeter was calibrated by using the Pre-calibrated end V-notch. The weir model was fitted carefully by using silicon. The passing flow through flume was controlled by either a valve downstream the pump or controlling the pump speed. The flow head over weir was measured at (4-5)H upstream weir model. In each run the discharge (Q), the bed level, the crest level, and the upstream water level were measured. The flow for all models was free flow as shown in Photo 2. Also, it should be noticed that there were some errors in measured variables in laboratory (±1-2%), which might be coupled with each others and affect the value of Cd.

ANALYSIS AND DISCUSSION OF THE RESULTS

Figure 1 shows the relationship between discharge coefficient (Cd) and relative head (H/Y) for rectangular sharp crested weir at different values of discharge. The Figure illustrates that the coefficient of discharge (Cd) is directly proportional to the relative head (H/Y) at a constant value of discharge. Fig. 2 shows the discharge coefficient for the sharp crested weir versus H/Y between the values 0.375 to 1.47. By using linear best fitting the experimental data is used in developing the following relationship

$$Cd = 0.6184 + 0.1078\ H/Y \qquad (12)$$

The developed formula could be used in computing discharge coefficient for the sharp crested weir as a function of relative head for H/Y ranged from 0.375 to 1.47 and $H/w > 2.4$. This was confirmed by Michael C. Johnson [10] in which he stated that for $H/w > 1.8$ the weir thickness had no effect on the value of Cd. Comparing the values of Cd resulted from the aforementioned formula and the corresponding values of Cd resulted from some other researchers as Rehbock [12], Kindsvater [8] and Rouse [10] showed that there was a congruency to a percentage ranged between 92% to 96% as illustrated in Fig. 3.

Figure 4 illustrates the relationship between coefficient of discharge (Cd) and relative head (H/Y) for different values of discharge on a broad crested weir. The figure shows that the coefficient of discharge is directly proportional to the relative head of the broad crested weir. This could be explained as the crest height increases the approach velocity decreases as shown in Fig. 5. The increase of crest height means a decrease in relative head, while the decrease of the approach velocity results in decreasing the total energy, E which decreases the value of Cd as illustrated in Eq. 6. From Fig. 5 the following relationship was developed, which could be used in calculating the discharge coefficient of the broad crested weir provided that $0.3 < H/Y < 1.5$, and $0.18 < H/w < 0.33$.

$$Cd = 0.5819 + 0.1677\ H/Y \tag{13}$$

A comparative study for the approach velocity correction coefficient (C_v) for a broad crested weir was carried out as shown in Fig. 7. From the figure it is found that the calculated values of Cv from the experimental data are highly consistent (98%) with that resulted from Bos formula [2].

The effect of relative height (H/Y) on discharge coefficient (Cd) of triangular V-notch at four values of head angle ($\theta=43^0, 36^0, 31^0, 27^0$) was considered. Figure 8 shows the relationship between coefficient (Cd) and relative height (H/Y) at different values of head angle and different values of flow discharge. The figure illustrates that the coefficient Cd is directly proportional to relative head. This could be explained due to the increase of relative head results from the decrease in crest height as shown in Fig. 9, while the decrease in crest height results in decreasing the flow head as shown in Fig. 10, which increases the discharge coeffecient value as illustrated in Eq. 14. Also, Fig. 11 concludes that the relative head is inversely proportional to the flow head. As the flow head decreases at constant values of both discharge and angle (θ), the coefficient Cd increases as illustrated in the following equation:

$$Cd = Q/[C(\tan \theta/2)(H^{5/2})] \tag{14}$$

Also, Fig. 8 shows that the coefficient Cd is inversely proportional to the head angle (θ) at constant value of relative head (H/Y). This could be attributed due to the fact that the increase of head angle means an increase in term ($\tan\theta/2$) in Eq. 14, which results in decreasing the value of the coefficient Cd. Although the decrease in head angle (θ) results in increasing the upstream head, the decrease in angle θ is more effective on discharge coefficient value than the increase in head as illustrated in Table 1.

Table 1. Head and coefficient Cd

Angle (θ)	Head (H) cm	Cd
43	6.7	0.605
36	7.1	0.634
31	7.45	0.655
27	7.8	0.675

From Fig. 8 it is found that the increase of relative head (H/Y) by 100% results in increasing the coefficient Cd by 6.3%, 7.5%, 8.0%, 8.4% at the head angle 43^0, 36^0, 31^0, and 27^0, respectively. The increase of head angle from 27^0 to 43^0 i.e. 60% decreases the coefficient Cd by 10%. It could be concluded that the head angle (θ) in V-notch is more effective on the value of discharge coefficient (Cd) than relative head (H/Y). This result is consistent with the studies carried out by Lenz [1], Barr [6] and the data taken from Kornil University, which concluded that the discharge coefficient of triangular weir depended mainly on the head angle θ.

Figure 12 shows the relationship between coefficient of discharge (Cd) and relative head (H/Y) for trapezoidal weir with side slope 1:4 for different values of flow discharge. From this figure, it was found that the increase of H/Y by 200% increased discharge coefficient (Cd) by 0.7% only, which could be neglected. It could be concluded that the effect of relative head (H/Y) on discharge coefficient (Cd) in case of trapezoidal weir with side slope 1:4 could not be considered. This could be attributed due to the shape of the weir, which compensate the effect of end contraction of the flow. This result is consistent with the results of USBR [14], in which they stated that the value of Cd of the trapezoidal weir with side slope 1:4 did not depend on the height of the crest.

CONCLUSIONS

From this experimental study the following points could be concluded:

1- For the sharp crested weir there is a directly proportional relationship between discharge coefficient, Cd and the relative head, H/Y.

2- For the rectangular sharp crested weir the following relationship was developed
$Cd = 0.6184 + 0.1078\ H/Y$, provided that $0.375 < H/Y < 1.47$.

3- There was a good agreement between the developed formula and the corresponding formulas developed by other researchers.

4- For the broad crested weir the coefficient of discharge was directly proportional to the relative head (H/Y).

5- The following relationship was developed for the broad crested weir coefficient provided that $0.3 < H/Y < 1.5$, and $0.18 < H/w < 0.33$.

$$Cd = 0.5819 + 0.1677\ H/Y$$

6- For the broad crested weir the values of $C_v = (H_e/H)$ resulted from the experimental study were consistent with Bos results [2] to a high ratio (99%).

7- For V-notch weir, although the coefficient of discharge was directly proportional to the relative head, it was found that it depended mainly on the head angle (θ).

8- Triangular V-notch was less affected by the relative head H/Y than both the sharp and broad crested weirs.

9- The effect of H/Y on the discharge coefficient (Cd) in case of trapezoidal weir with side slope 1H:4V could not be considered, which consistent with the results of USBR [14].

REFERENCES

1- Arno T. Lenz “Viscosity and surface tension effect on V-notch weir coefficient” Trans. ASCE, Vol. 108, 1943 (Source Terry [13]).
2- Bos, M. G. “ Discharge measurement structures” ILRI Publication 20, 3rd Revised Edition, Wageningen, the Netherlands, 1988. (Source Terry W. Sturm “Open channel hydraulics” Mc. Graw Hill, International Edition, New York, 2001).
3- Chow, Ven Te “Open channel hydraulics”, New York, Mc. Graw Hill, 1959.
4- El-Alfy, Kassem S. “Effect of Geometry of the broad crested weir on the dimensions of the downstream apron”, Minufiya Eng. Res. Bulletin, Vol. 22, No. 1, January 1999.
5- Henderson, F. M. “Open channel flow”, Macmillan Publishing Co., Inc., New York, 1966.
6- James Barr, “Experiments upon the flow of water over triangular notches”, Engineering Apr. 8-15, 1910 (source [12]).
7- J. B. Francis “Lowell hydraulic experiments”, 4th ed. (New York : D.Van. Nostrand Company, Inc. 1883 (source [12]).
8- Kindsvater, C. E., and R. W. C. Carter “Discharge characteristics of rectangular thin-plate weirs”, J. Hyd. Div., ASCE, 83, No. HY6, 1957, pp 1453-1 to 35. (source [12]).
9- M. C. Johnson “Discharge coefficient analysis for flat-topped and sharp crested weirs” Irreg. Sci. bulletin, 2000.
10- Rouse, H. and Le Reid “Model research on spillway crests”, Civil Engineering, Vol. 5, 1935. (source F. M. Henderson, “Open channel flow”, Macmillan publishing co., Inc., New York, 1966).
11- Robrett and Jozeff “Fluid mechanics and its engineering application” McGraw Hill Co., New York.
12- T. Rehbock “Discussion of precise weir measurements” by E.W. Schoder and K.B. Turner, Trans, ASCE, Vol. 93, pp1143, 1929.
13- Terry W. Sturm “Open channel hydraulics” McGraw Hill, International Edition, New York, 2001.
14- USBR, U.S. Dept. of the Interior, Bureau of Reclamation “Water measurement manual” 1997, Denver, Colorado, USA.

NOTATION

b	crest length.
B	approach channel width.
C_c	contraction coefficient.
Cd	coefficient of discharge.
Cd_e	effective discharge coefficient which depends on H/Y and b/B.
C_v	velocity coefficient.
E	flow energy.
E_c	critical value of flow energy.
H	head on weir crest.
H_e	effective head.
k_L	constant.

k_H	constant = 0.001m.
L	crest length.
N	number of end contractions.
Q	discharge.
q	discharge per unit length.
R	mean hydraulic radius.
R_e	Reynolds No.
v_a	approach velocity.
w	crest thickness.
We	Weber number = $V/(\sqrt{\sigma}/\rho L)$.
Y	crest height over bed.
y_c	critical depth.
θ	head angle of triangular weir.

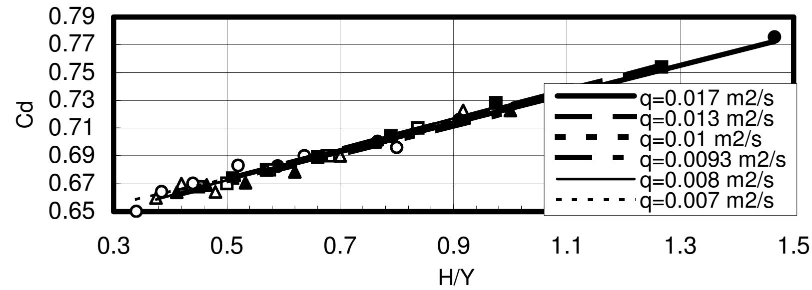


Fig. 1 The relationship between discharge coefficient and relative head for sharp crested weir at different values of discharge.

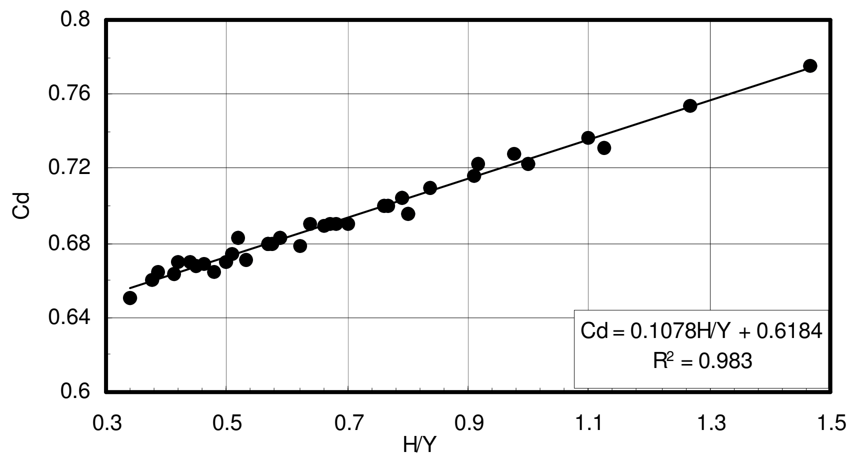


Fig.2 The relationship between discharge coefficient and relative head for sharp crested weir .

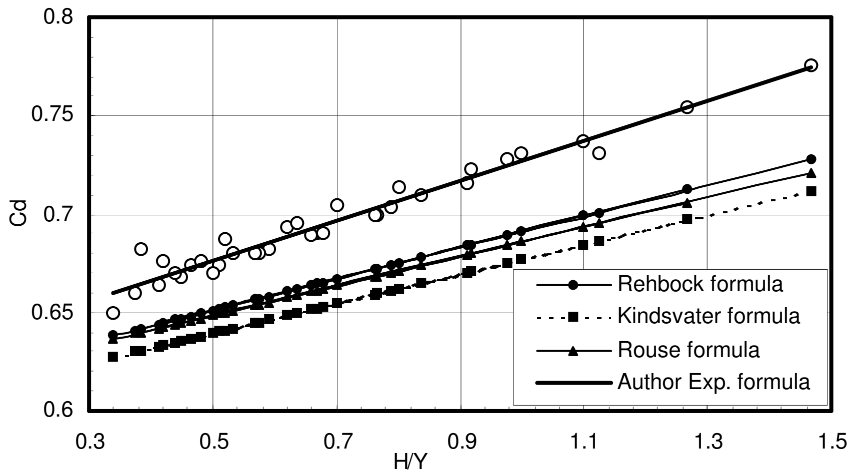


Fig. 3 A comparison among the present experimental study and some other studies on sharp crested weir .

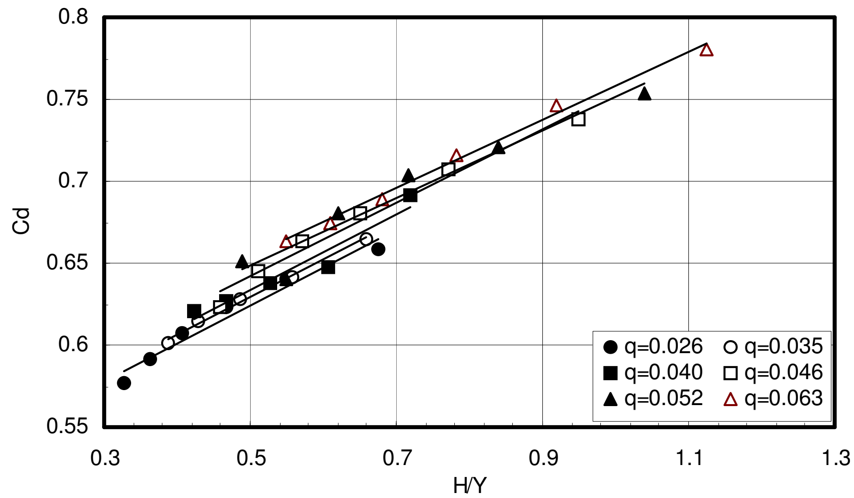


Fig. 4 The relationship between discharge coefficient and relative head at different values of discharge for broad crested weir.

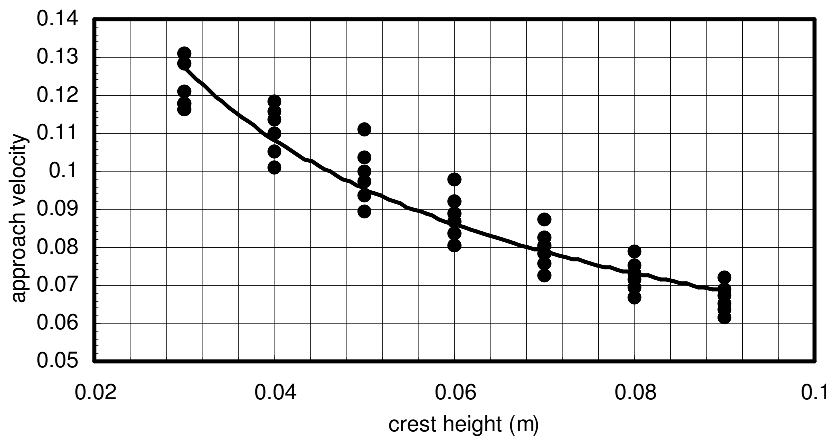


Fig. 5 The relationship between the approach velocity and the relative head on the broad crested weir .

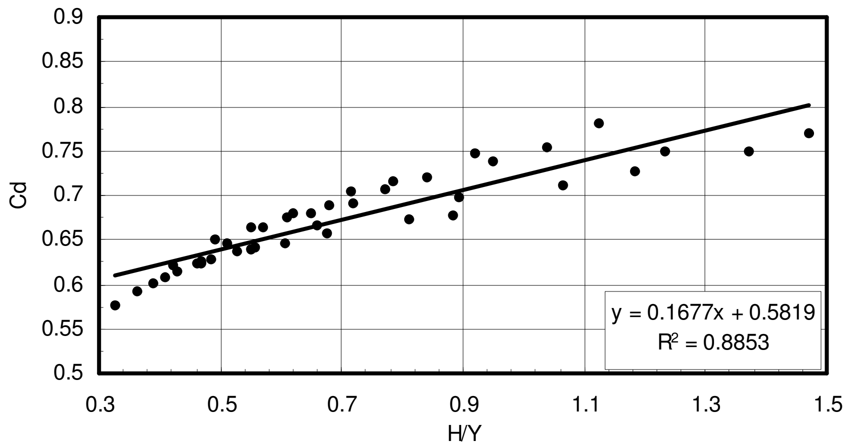


Fig. 6 The relationship between discharge coefficient and relative head at different values of discharge for broad crested weir.

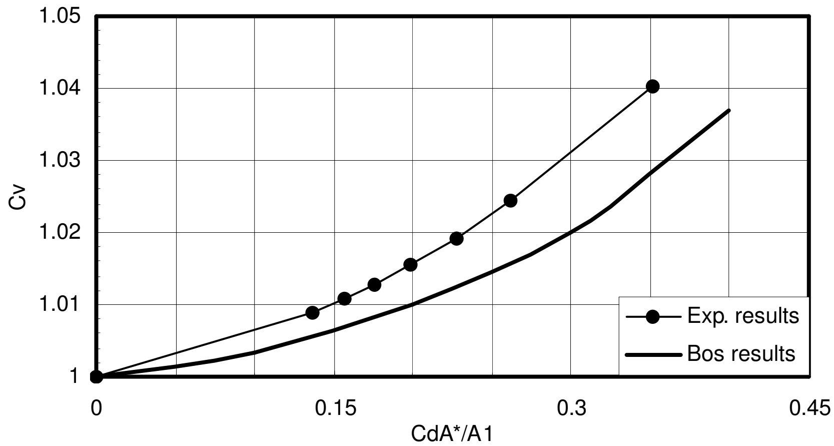


Fig. 7 a comparison between approach velocity correction coefficient for a broad crested Weir.

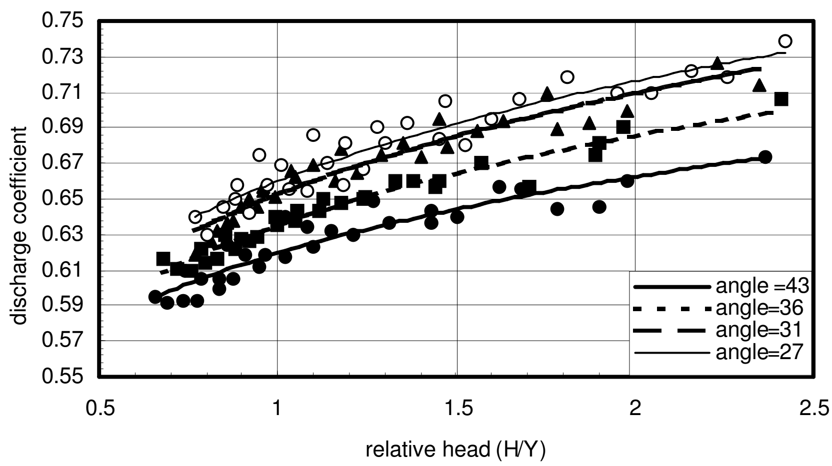


Fig. 8 The relationship between discharge coefficient and relative head at different values of head angle of V-notch weir.

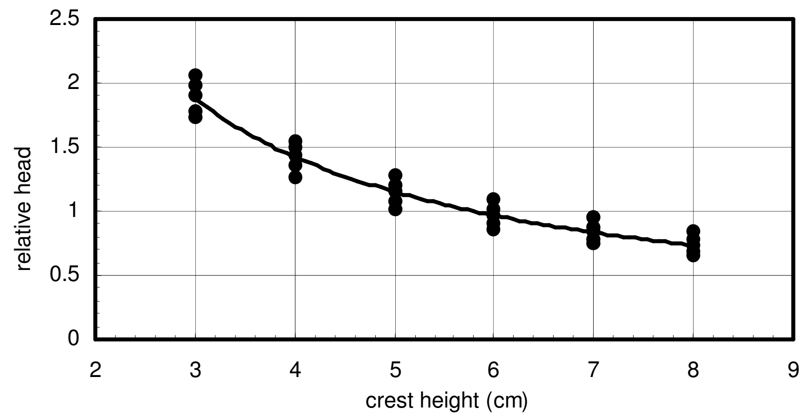


Fig. 9 The relationship between the relative flow head and crest height over V-notch weir.

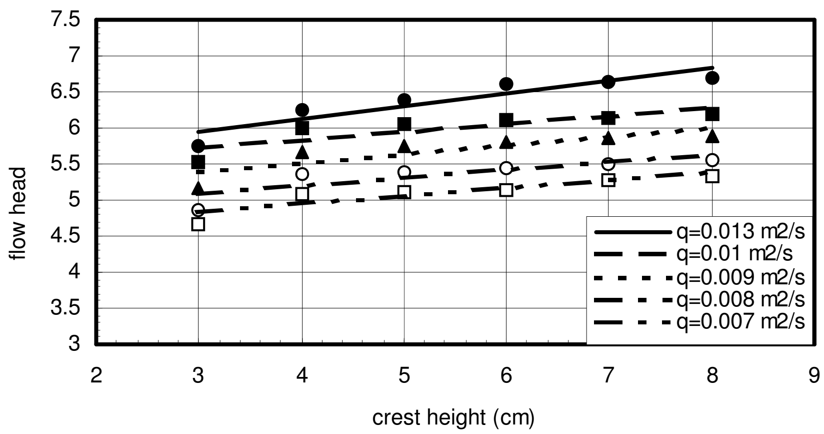


Fig. 10 The relationship between flow head and crest height over V-notch weir at θ=43.

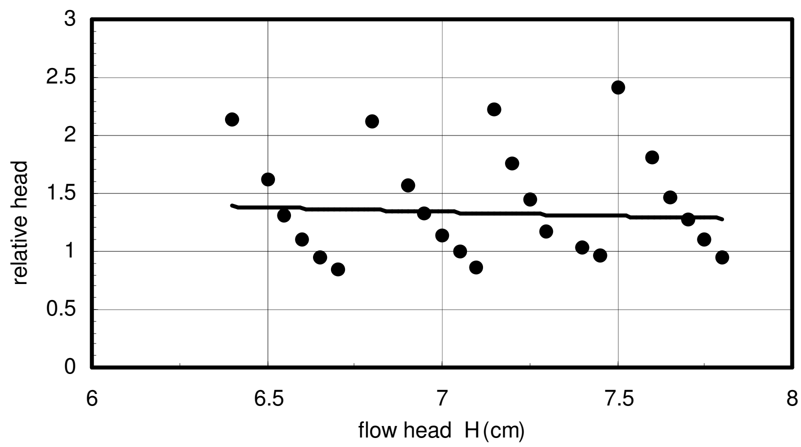


Fig. 11 The relationship between the relative head and the flow head of triangular weir at θ=43.

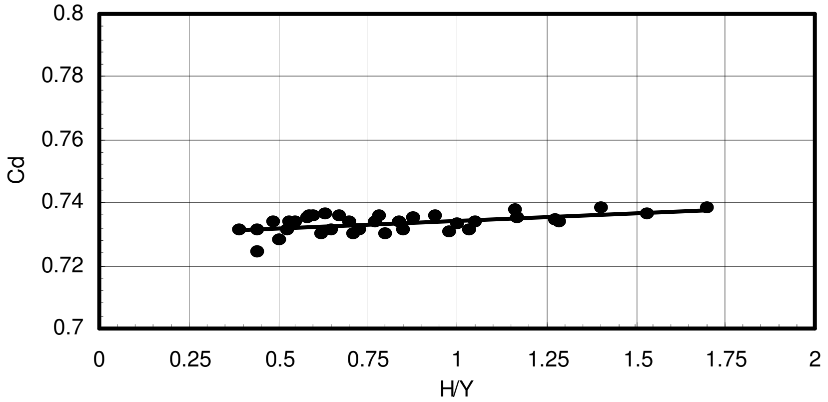
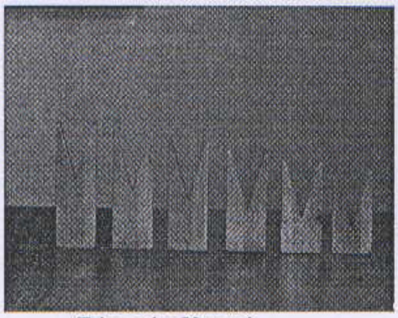
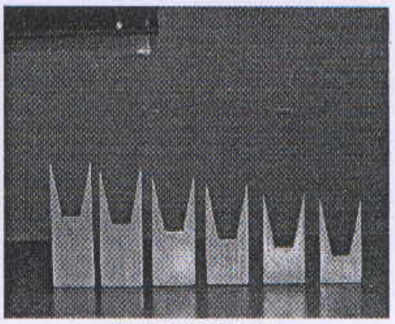


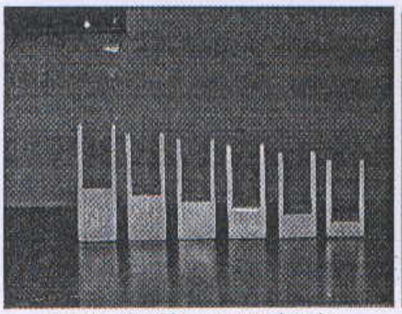
Fig. 12 The relationship between the coefficient of discharge
and the relative head for the trapezoidal weir



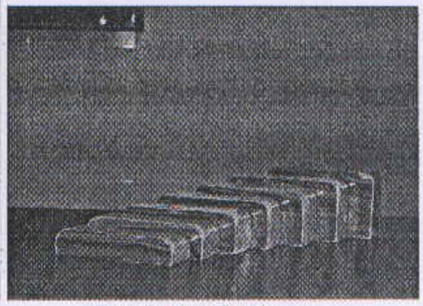
Triangular V-notch.



Trapezoidal weir.



Rectangular Sharp crested weir.



Broad crested weir.

Photo. 1 Some models of different weirs.

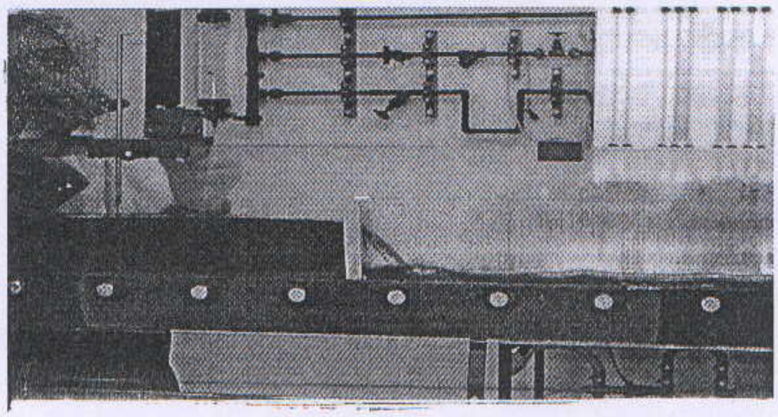


Photo. 2 Flow over rectangular sharp crested weir